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C · B

Código **matemático**
unificado
para Iberoamérica

Edita

Unified Mathematical Code for Latin America



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Index

Introduction.....	7
Representation agreements	9
Chapter 1. Literal characters	12
Distinctive prefixes	12
1.1. Latin characters	12
1.2. Greek characters	14
1.3. Typographic variants of the Latin alphabet	15
<i>First typographic variant</i>	15
<i>Second typographic variant</i>	17
Chapter 2. Figures and Numbers	18
2.1. Figures or numbers.....	18
2.2. Integers.....	18
2.3. Decimal numbers	19
2.4. Fractional numbers.....	19
2.5. Special Representations	20
a) <i>Special numbers</i>	20 b)
<i>Numbers that include letters with numerical value</i>	20
c) <i>Numbers with figures in typographic or color variants</i>	21
d) <i>Numbers indicating the numbering base</i>	22
2.6. Representation of numerical sets.....	22
Chapter 3. Delimiters.....	23
3.1. Horizontal delimiters	23
3.2. Vertical delimiters	25
3.3. Two-dimensional structures (matrices, determinants and tables).....	27
3.3.1. Explicit two-dimensional representation.....	28
3.3.2. Semi-linearized Braille representation.....	34
3.4. Delimitation of regions within a two-dimensional structure	38
3.4.1. <i>Regions in two-dimensional structures represented in Braille in the form explicit</i>	39
3.4.2. <i>Regions in two-dimensional structures represented in Braille in the form semi-linearized</i>	42
Chapter 4. Indexes and Marks.....	46
4.1. Positions	46
4.2. Indices.....	46
4.2.1. General case.....	46



4.2.2. Numerical subscripts.....	48
4.3. Brands	48
4.3.1. Right-hand superscript marks	48
4.3.2. Superscript marks	50
4.3.3. Trademarks in the undersigned	51
4.3.4. Marks in subscript on the right or left or superscript on the left	52
4.4. Symbols affected by multiple indices and/or marks.....	52
4.4.1. Writing order	52
4.4.2. Shifted Indexes	53
Chapter 5. Arithmetic and algebraic operators.....	55
5.1. Elementary arithmetic operations	55
5.2. Powers and roots	57
5.2.1. Powers.....	57
5.2.2. Roots.....	58
5.3. Combinatorics	59
5.4. Other operators	60
5.5. Operators on sets of numbers.....	60
5.5.1. Summation or summation	60
5.5.2. Producer	61
5.5.3. Co-producer	61
5.6. Operators on two-dimensional structures.....	62
Chapter 6. Arithmetic and algebraic relationships.....	64
6.1. Relationships	64
6.2. Denied relationships.....	65
Chapter 7. Set Theory	67
7.1. Basic concepts	67
7.2. Operations between sets	68
7.3. Relationships between the elements of a set	69
7.4. Cardinals	70
Chapter 8. Logic.....	71
8.1. Quantifiers.....	71
8.2. Propositional Logic.....	71
Chapter 9. Analysis and Calculation	73
9.1. Generalities.....	73
Writing Ordered Pairs	74
Piecewise Function	76
Intervals	76



9.2. Limits	77
9.3. Derivatives and differentials	79
<i>Derivative of a function</i>	79
<i>Derivative of order n</i>	80
Partial derivative	80
<i>nth partial derivative</i>	81
<i>Nabla, Laplacian, d'alambertian</i>	83
9.4. Integrals.....	83
Chapter 10. Classical Functions	85
10.1. Sequences, progressions and series	85
10.1.1. Successions.....	85
10.1.2. Progressions.....	86
10.1.3. Series.....	86
10.2. Functions represented by symbols/literal notations	86
10.2.1. <i>Exponential and logarithmic functions</i>	86
10.2.2. <i>Trigonometric (circular) functions</i>	87
10.2.3. <i>Hyperbolic Functions</i>	89
10.2.4. <i>Other functions, parameters and characteristics represented by symbols</i>	90
Chapter 11. Geometry and Vectors	91
11.1. Geometric elements.....	91
11.2. Vectors.....	92
11.3. Geometric Relationships.....	93
11.4. Operations between geometric elements.....	94
11.5. Angle Measurements.....	95
11.6. Polygons and curves	96
Chapter 12. Statistics and Probability	97
Chapter 13. Financial Mathematics	100
Chapter 14. Adaptation and editing criteria	102
14.1. Aspects to take into account when writing mathematical texts	102
14.2. Partitioning Mathematical Expressions.....	103
14.3. Other aspects to consider	106
14.4. Highlighting and coloring elements and expressions	108
14.5. Fill-in spaces within mathematical expressions.....	109
Appendix 1. Mathematical and physical-chemical units	111
1.1. Fundamental physicochemical units	111
1.2. Derived physicochemical units.....	112
1.3. Prefixes for multiples and submultiples.....	113



Appendix 2115

2.1. Ordinals.....115

2.2. Roman Numerals115

2.3. Coins115

Appendix 3. Combinations of arrows and points117



Introduction

In all areas of knowledge, new content and forms of communication have emerged. represent them. Access to all this information is a universal and unrestricted right.

Over the years, the need to represent mathematical notations and expressions not contemplated in the initial version of the Code has arisen.

Unified Mathematical Framework (CMU) of 1987 (statistics, financial mathematics, two-dimensional representations, some delimiters, typographic resources that were not used in those years, etc.) or appeared later in the general educational or scientific field.

Braille is a living organism that, in recent decades, has demonstrated an astonishing vitality. It remains unfinished, displaying a potential that honors its inventor.

In particular, Braille signography in mathematics and other scientific disciplines is an open, flexible, and dynamic tool, capable of meeting the challenges and continuous changes in the graphic representation of mathematical expressions.

On the other hand, inclusive education has meant the participation of a greater number of students with visual impairments at both intermediate and higher levels superiors, which has motivated the need to update the representation of Some of the expressions used in the mathematics classroom. It is a signography which, of course, covers different educational stages and allows for the representation of symbols at both elementary and higher levels.

This social and educational dynamic led to the Ibero-American Braille Council (CIB), formed by representatives of the Spanish and Portuguese speaking countries, to consider the need for an update of the Unified Mathematical Code and to decide, for this purpose, the formation of a Technical Commission of Sciences, dedicated to the A broad and in-depth study of mathematical content, its representations, and correct application using Braille. This update to the CMU is the result of this commission's work.

For all the above reasons, this code and its braille representation standards are subject to new extensions and modifications that may impose their use in the future. application and evolution of educational and scientific processes.

Based on the validity and consistency of most of the signs approved in 1987, the final result of this update has been the incorporation of new signs and representations, as well as some modifications that are considered necessary.

A new feature of the 2023 version of the CMU is its publication in Spanish and Portuguese.

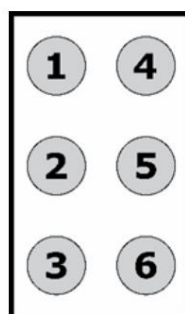


The code update has been done taking into account:

- What is the reading and writing tool for students who use Braille?
within the novelty that the widespread reality of "inclusive education" represents.
- The needs of mathematics professionals who use Braille.
- The needs of students who use Braille, as detected by the teachers in their daily work in the mathematics classroom. A good use
The use of the code makes writing easier for blind students and avoids some errors observed in Braille reading.
- New technologies, which are increasingly present in the classroom and in the education of blind people.
- The current and future demands of digital editing and transcription, which, to produce higher-quality Braille texts, require the use of an appropriate code.
- That transcription guidelines have been given that have allowed us not to be exhaustive in the enumeration of specific representations.
- The graphic representation of visual symbols has been taken into account as relative positions, except in some widely used signs, such as the addition sign or equality, in which meaning is taken into account.

Throughout the chapters of this new edition of the Unified Mathematical Code, It refers to the series into which the combinations formed with are divided. the six points of the Braille generating sign.

The allocation of dots to represent a braille character is:



For example, the braille character that represents the letter z is made up of the points 1, 3, 5 and 6: z



The seven series are listed below:

1st Series: points 1, 2, 4 and 5 are used.

abcdefghijklmnopqrstuvwxyz

Series 2: Point 3 is added to the combinations of the 1st series.

klmnopqrst

Series 3: Point 6 is added to the combinations of the 2nd series.

uvwxyz & \ á é ú

Series 4: Point 6 is added to the combinations of the 1st series.

1 2 3 4 5 6 7 8 9 w

5th Series: Move down one position to the points of the 1st series, points 2, 3, 5 and 6.

, ; : * ? + = < } >

Series 6: points 3, 4, 5 and 6.

í ì ñ ò - .

Series 7: points 4, 5 and 6.

' ^ % " _ @ {

Representation agreements

Throughout the chapters that make up this code, the signs will be presented in tables with a series of columns: visual sign, sign in Braille, number of the Braille dots that form the sign and meaning.

In the respective column, the enumeration of the braille dots that form each sign complies with the following conventions:

- The Braille sign is represented by a number formed by the digits of each of the points that compose it.

Example:

p braille dots: 1234

- When a braille character consists of more than one cell, the dots of each braille character are separated from the previous and next character by a hyphen.



Example:

{n braille dots: 46-1345

- When the blank space is part of a sign it is represented by 0.

Example:

Braille dots : 0-123-0

- A "blank semi-braille box" is defined as one of the columns of the braille box without any dots.
- When a sign requires a blank semi-enclosure to follow (a its right) is represented as 0d.

Examples of the sign 456-0d. Note that, in this first case, the box that follows does not contain any of points 1, 2 or 3:

%{p braille dots: 456-0d-46-1234

However, in this other case it does have them, so it is necessary to leave a box without any period between both characters:

% p braille dots: 456-0-1234

- When a sign must be preceded by a blank semi-enclosure (to its left) it is represented as i0.

Examples of the i0-123 sign. Note that, in this first case, the previous box does not contain any of the points 4, 5, or 6:

kl braille dots: 13-10-123

However, in this case it does have them, so it is necessary to leave a box without any period between both characters:

#l braille dots: 3456-0-123

- Throughout the text, reference is sometimes made to "signs or numbers in "low position". This refers to all the signs of the 5th series.

Numerous examples are included to clarify the use of the signs.

The code is structured into 14 chapters and three appendices. It is recommended that you read Chapter 14, *Adaptation and Editing Criteria*, before using the code.



Some highlights of the code include:

1. News.

- The distinction of colors and changes in typography are contemplated within numerical and mathematical expressions (chapters 1 and 2).
- The representation of fractional numbers has been modified (*Chapter 2*).
- Added horizontal representation of other delimiters (*Chapter 3*).
- Differentiated representation of two-dimensional expressions (systems, matrices, determinants, etc.) in developed and semi-linear formats (*Chapter 3*).
- Signs for statistics and financial mathematics are collected in their most common notations (chapters 12 and 13).
- Issues related to the adaptation to Braille of new typographic resources used in the edition of current textbooks are also considered.
which are often used for educational purposes, such as the Braille representation of "blank" or "fill-in" spaces, and underlined, crossed-out, and colored expressions or characters and signs (*Chapter 14*).
- Rules are established for the partitioning of mathematical expressions that exceed the number of characters in braille printing (*Chapter 14*).
- Abbreviated and alternative representations are included that allow for some flexibility for note-taking by students or for handling in editors (trigonometric, limits, derivatives...) (chapters 9 and 10).
- Braille representations of "physicochemical units" (or "algebras of quantities") are included following the international agreements contained in the 9th edition of 2019 of the International System of Units (*Appendix 1*).

2. Recommendations for Braille editions.

- It is recommended (primarily to editors) that science texts include tables with the symbols used and their meaning.
- It is important to make known the function and use of auxiliary parentheses when they appear for the first time in a text, since they are a resource specific to the Braille system (*Chapter 3*).
- When a mathematical expression appears within a text, two spaces must be left before and two after (*Chapter 14*).
- To avoid possible confusion, it is recommended not to use shorthand Braille in texts on exact or natural sciences.



Chapter 1. Literal characters

Distinctive prefixes

The Unified Mathematical Code approved in Montevideo (Uruguay) in June 1987 introduced the concept of "alphabetic prefixes." This meant that, based on lowercase Latin letters, each of these prefixes characterized them as letters of an alphabet, according to the sign established to distinguish each of them. Lowercase Latin letters also had their prefix, since it was necessary,

example, avoiding confusion between numbers and letters.

This concept remains the same in substance and form, but some clarifications are necessary.

- A) Except for lowercase Latin letters, in which the prefix is omitted except
To avoid confusion, each letter (regardless of the alphabet) must be considered a Braille symbol consisting of two inseparable characters —*prefix + base letter*— and treated like any other symbol that occupies two cells. This means that, in mathematical expressions, no prefix (not even that of the lowercase Latin letters "a" to "j" within numerical expressions) will be valid for several consecutive letters.
- B) Since the function of "alphabetic prefix" is not the only one performed by the Braille signs chosen for this purpose (precisely, they are quite commonly used for brands), care must be taken not to create confusion, applying them to the lowercase Latin letters a correct use of point 5 when used as «Latin minuscule prefix».
- C) From what has been said it follows that in numerical expressions that contain letters, in which not all of them always go at the end, all the braille characters of the 1st series not preceded by point 5 have a numerical value. That is, they are numbers.

Examples:

234a5 #bcd@ae

234ae #bcd@a@e

In addition to what is expressed later in this chapter, other examples of the use of point 5 will be discussed in detail in the corresponding chapters.

1.1. Latin characters

Only the letters of the Latin alphabet used in mathematics are included.



Lowercase letter	Braille sign	Braille dots	Capital letter	Braille sign	Braille dots
to		1	TO	{a	46-1
b		12	B	{b	46-12
c		14	C	{c	46-14
d		145	D	{d	46-145
and		15	AND	{e	46-15
F		124	F	{f	46-124
g		1245	G	{g	46-1245
h		125	H	{h	46-125
i		24	I	{i	46-24
j		245	J	{j	46-245
k		13	K	{k	46-13
l		123	L	{l	46-123
m		134	M	{m	46-134
n		1345	N	{n	46-1345
either		135	EITHER	{o	46-135
p		1234	P	{p	46-1234
q		12345 Q		{q	46-12345
r		1235	R	{r	46-1235
s		234	S	{s	46-234
t		2345	T	{t	46-2345
or		136	OR	{u	46-136
v		1236	V	{v	46-1236
w		2456 W		{w	46-2456
x		1346	X	{x	46-1346
and		13456	AND	{y	46-13456
z	abcdefghijklmnopqrstuvwxyz	13456		{z	46-1356



In case lowercase letters may cause confusion when forming part of

signs or expressions, must be preceded by the character @ (5), called "prefix" from Latin minuscule».

The Braille character { (46), which is an inseparable part of the Latin letters capital letters, it is called "Latin capital letter prefix".

1.2. Greek characters

Name	Lowercase lettersign	Braille sign	Braille dots	Capital letter	Braille sign	Braille dots
alpha	a	'a 4-1 'b 4-12 'g		A	^a 45-1 ^b 45-12 ^g	
beta	ÿ	4-1245 'd 4-145 'e		B	45-1245 ^d 45-145 ^e	
gamma ÿ		4-15 'z 4-1356 '5 4-156		ÿ	45-15 ^z 45-1356 ^5	
delta	ÿ	'4 4-1456 ÿ 'i 4-24 'k		ÿ	45-156 ^4 45-1456 ^i	
epsilon ÿ		4-13 'l 4-123 'm		E	45-24 ^k 45-13 ^l	
dset	ÿ	4-134 ÿ 'n 4-1345 'x		ÿ	45-123 ^m 45-134 ^n	
and	ÿ	4-1346 'o 4-135 'p		H	45-1345 ^x 45-1346 ^o	
zeta	ÿ	4-1234 'r 4-1235 's 4-234 't			45-135 ^p 45-1234 ^r	
iota	...	4-2345 'u 4-136 'f		I	45-1235 ^s 45-234 ^t	
kappa	k	4-124 ÿ		K	45-2345 ^u 45-136 ^f	
lambda ÿ				L	45-124	
my	ÿ					
noether	n			N		
xi	ÿ			ÿ		
omicron ÿ				O		
pi	ÿ			Pi		
ro	ÿ			P		
sigma	ÿ ÿ			ÿ		
tau	ÿ			T		
upsilon ÿ				I		
fi	ÿ					



Name	Lowercase letter	Braille sign	Braille dots	Capital letter	Braille sign	Braille dots
ji	ÿ	'& 4-12346 ÿ 'y 4-13456 ÿ 'w			^& 45-12346 ^y 45-13456	
psi	ÿ	4-2456 ÿ			^w 45-2456	
omega ÿ						

The character ' (4), which is an inseparable part of the lowercase Greek letters, is called "Greek minuscule prefix".

The character ^ (45), which is an inseparable part of the Greek capital letters, is called "capital Greek prefix".

Other commonly used Greek characters that have a different prefix are:

Name	Visual sign	Braille sign	Braille dots
varzeta	ÿ	'ó4	4-346-1456
varepsilon	ÿ	'óe	4-346-15
varro	ÿ	'ór	4-346-1235
varpi	ÿ	'óp	4-346-1234
varsigma	ÿ	'ós	4-346-234
varfi	ÿ	'óf	4-346-124
Varkappa	ÿ	'ók	4-346-13
digamma	ÿ	%v	456-1236

1.3. Typographic variants of the Latin alphabet

Different fonts, whether because they belong to another character set (as is the case with Gothic letters) or because different typographic resources are used in their representation (such as colors, shading effects, outlines, etc.), will require the following resources for their representation in Braille.

First typographic variant

The character _ (6), which is an inseparable part of this first typographic variant, It is preferably used to differentiate lowercase letters with a typography distinct from the base; in particular, the Gothic letter.



The character " (56), which is an inseparable part of this first typographic variant, It is preferably used to differentiate capital letters with a typography distinct from the base; in particular, the Gothic letter.¹

The following table uses the well-known character set in Unicode as "Mathematical Bold Fraktur", simply as an example.

Character Latin	Letter lower case	Braille character	Points braille	Letter capital	Braille character	Points braille
a/A		_a 6-1 _b 6-12			"a 56-1 "b 56-12	
b/B		_c 6-14 _d 6-12			"c 56-14 "d 56-12	
DC		6-145 _e 6-15 _f 6-124			56-145 "e 56-15	
d/D		_g 6-1245 _h 6-12			"f 56-124 "g 56-12	
e/E		6-125 _i 6-24 _j 6-13			56-1245 "h 56-12	
f/F		_j 6-245 _k 6-13			56-125 "i 56-24 "j 56-13	
g/G		_l 6-123 _m 6-134			56-245 "k 56-13 "l 56-12	
h/H		_n 6-1345 _o 6-135 _p 6-12			56-123 "m 56-134	
i/I		6-1234 _q 6-12345 _r 6-12			"n 56-1345 "o 56-13	
j/J		6-1235 _s 6-234			56-135 "p 56-1234	
k/K		_t 6-2345 _u 6-12			"q 56-12345 "r 56-12	
l/L		6-136			56-1235 "s 56-234	
m/M					"t 56-2345 "u 56-12	
n/N					56-136	
or/or						
p/P						
q/Q						
r/R						
H.H						
t/T						
u/U						

¹ See chapter 14 for the use of 56 as a space-filling symbol.



Character Latin	Lowercase letter	Braille character	Braille dots	Capital letter	Braille character	Braille dots
v/V		_v 6-1236	_w 6-2456		"v 56-1236	"w
w/W		_x 6-1346	_y 6-13456		56-2456 "x	56-1346
x/X		_z 6-1356	ÿ		"y 56-13456	"z
and/and					56-1356	
z/Z						

Second typographic variant

If, in addition, it is necessary to distinguish another typographic variant, the inseparable prefix or (346) for lowercase letters and 3 (146) for uppercase letters.

Note: in the edition of mathematical expressions in Braille, for literal characters and their fonts only the signs collected in this chapter will be used, unless in other chapters another form of representation is provided for which an explanatory note must be attached and what is established in chapter 14 regarding *Criteria for adaptation and edition* or the indications that could be collected as «Braille transcription notes».



Chapter 2. Figures and numbers

2.1. Figures or numbers

They are represented by the braille characters of the first series preceded by # (3456).

Visual sign	Braille sign	Braille dots	Meaning
1	#a	3456-1	one
2	#b	3456-12	two
3	#c	3456-14	three
4	#d	3456-145	four
5	#e	3456-15	five
6	#f	3456-124	six
7	#g	3456-1245	seven
8	#h	3456-125	eight
9	#i	3456-24	nine
0	#j	3456-245	zero

2.2. Integers

The # sign (3456) acts as an indicator for all digits in the number, so only appears at the beginning of the amount.

Whenever there is a separation between groups of figures in visual characters, in Braille, point 3 will be used as a separator.

Examples:

31,720 #ca.gbj

3 802 197 #c.hjb.aig

46781 #dfgha

Negative numbers will be represented having as first braille character the sign - (36).²

² The sign-(36) corresponds to the minus sign (see chapter 5. *Arithmetic and algebraic operators*).



Examples:

-7 -#g

-29 -#bi

-25 347 -#be.cdg

2.3. Decimal numbers

The decimal part separation sign in Braille will be represented by the element , (2), even when other signs appear in visual characters, such as the period.

Examples:

3.2 #c,b three integers, two tenths 3.2 #c,b

three integers, two tenths 3.2 #c,b three

integers, two tenths

In recurring decimal expressions, the beginning of the period will be transcribed by means of point 2, regardless of its representation in visual characters. If it is considered necessary to highlight the representation of this period, you can go to resource of auxiliary parentheses³ after point 2 that marks the beginning of the period.

Examples:

3,254 #c,b,ed

7,29 #g,,bi

3,73456345 #c,gcdef,{cde}

Expressions with ellipses in the decimal part will be written with... (3-3-3):

3,1415... #c,adae...

2.4. Fractional numbers

This section deals only with whole fractions or fractional numbers.

³ On the use of auxiliary parentheses, see chapter 3.



$$\frac{3}{4} \text{ or } \frac{3}{4}$$

The numerator, preceded by a numerical sign, will be written at the top of the box (1st series). The denominator, below, without a numerical sign, in the upper part lower (5th series), followed by a blank box.

Examples:

$$\frac{3}{4} \text{ or } \frac{3}{4} \quad \#c^* \quad \text{Three quarters}$$

$$\frac{38}{63} \quad \#ch+: \text{Thirty-eight sixty-thirds} \quad \#c\#a? \text{ Three whole}$$

$$3\frac{1}{5} \quad \text{numbers and one-fifth}$$

2.5. Special performances

a) Special issues

Some numbers have special representations, such as:

- The number pi: $\dot{y}$ 'p (4-1234) • The number e (base of natural logarithms): e e (15)
- The number i (imaginary unit): ii (24)

Examples:

$$\dot{y}=3.141592... \text{ 'p}=\#c,adaeib...$$

$$e=2.718281... e=\#b,gahbha...$$

$$1+2i \quad \#a+\#b@i$$

In this example, you can see that the dot 5 @ has been used before the letter "i", which represents the imaginary unit, to indicate that it is a lowercase Latin letter of the first series rather than a number.

b) Numbers that include letters with numerical value

The # sign (3456) acts as an indicator for all digits in the number, whether they are numbers or are represented by letters (as occurs with numbers in bases greater than 10), so it only appears at the beginning of the quantity.



The letters are represented as indicated in chapter 1 and, in this case, the

Latin lowercase letters from "a" to "j" will be preceded by the Braille character @ (5) to indicate its status as a lowercase Latin letter.

Examples:

1B4D #a{bd{d

1b4d #a@bd@d

14bd #ad@b@d

b1d4 #@ba@dd

1axb #a@ax@b

c) Numbers with figures in typographic or color variant

When there is a typographic or color variation in numbers with character significant, they will be transcribed by preceding the numerical sign with the prefix " (56).

Example:

24 "#bd

In case a second typographic or color variant appears, the sign 3 (146).⁴

In the event that typographic or color variations occur in certain figures of the same number, the corresponding indicator will immediately precede the respective figure.

The text in which this situation is presented must necessarily include a note clarification.

Example: 3429, where the number 4 is red and the number 9 is green. The text where this number appears distinction will begin with a note:

«In the following, the sign 3 (146) indicates that the character

located immediately to its right is red, and the sign " (56) will indicate that the character located immediately to your right is green»:

#c3db"i

⁴ As can be seen in chapter 1. *Literal characters*, section 1.3, both " and 3 correspond with some of the prefixes for typographic variants of letters.



d) Numbers with indication of the numbering base

In the visual representation of numbers in different bases, this is usually written in subscript to the right after the digits that make up the number.

In Braille, the right subscript sign is represented by í (34) (see chapter 4. *Indexes and marks*) and is written after the digits of the number and before the figure indicating the base.

1012 #ajaí#b Number 101 in base 2

2.6. Representation of numerical sets

Visual sign	Braille sign	Braille dots	Meaning
ℕ	%n	456-1345	natural numbers
ℤ	%z	456-1356	whole numbers
ℚ	%q	456-12345	rational numbers
ℝ	%r	456-1235	real numbers
ℂ	%c	456-14	complex numbers
ℍ	%h	456-125	quaternions
℔	%o	456-135	Cailey numbers (octonions)
ℙ	%p	456-1234	prime numbers

Example:⁵ "The set of real numbers, ℝ, is formed by..." is transcribed in Braille:

<{the set of numbers
real, %r, is made up of...<

Other special representations can be expressed in Braille by going to the typographic variants considered in chapter 1. For example, for numbers irrational could be:

"i (56-24)

I "i (56-24)

⁵ For writing mathematics text within literary text see chapter 14.



Chapter 3. Delimiters

3.1. Horizontal delimiters

"Horizontal delimiters" are considered to be those graphic resources that separate or unify written expressions one after the other, in the same line. For example, a numeric range.

Visual sign	Braille sign	Braille dots	Meaning
(	2	126	open parentheses
)	̀	345	close parentheses
[	á	12356	open brackets
]	ú	23456	close brackets
{	@l	5-123	open keys
}	%,	456-2	close keys
˘	@k	5-13	open angle brackets
˘	{,	46-2	close angle brackets
˘	@á	5-12356	open class brackets
˘	ú,	23456-2	close class brackets
(	@̀	5-345	open special keys
)	2,	126-2	close special keys
	%˘	456-0d	vertical bar
˘		456-123	double bar
Braille specific		26	open auxiliary parentheses
Braille specific		35	close auxiliary parentheses
/		6-2	slash
\		5-3	backslash
,		0-2	separating comma
.		0-3	separation point
;	%l ? } _ , @	0-23 ;	separating semicolon



Visual sign	Braille sign	Braille dots	Meaning
ȳ	á,	12356-2	open roof
ȳ	@ú	5-23456	close roof
ȳ	á.	12356-3	open ground
ȳ	_ú	6-23456	close floor
:	@,	5-2	colon
-	-	0-36-0	short script
---	---	0-36-36-36-0 ?	long dash
?		0-26-0	question

Note:

- *Ceiling function* is the one that assigns to each real number the highest integer next to excess, that is, the smallest whole number equal to or greater than that real number.
- *Floor function* or *integer part* is the one that assigns the largest value to each real number. integer equal to or less than that real number.

Note: The appearance of the space preceding the comma, period, semicolon and question mark is justified by the need to avoid confusion with the fractional numbers.

Examples of some horizontal delimiters:

Order the numbers from least to greatest

$$\frac{3}{23} \text{ , } 1, 4, 7, 8, 0, \frac{1}{-}, \text{ , } 3, 2$$

{order the numbers #c; , -#a , #d , #g, h , #j , #a: , 'p

, #c, b

The scale is one thirty thousand,
1:30000

{the scale is one thirty thousand,
#a@, #cjjj



Note: Auxiliary parentheses do not have their corresponding ink sign.

They constitute a resource specific to Braille to limit certain expressions that, in the visual writing, appear unified in various ways, such as the different size, different level with respect to the basic line of writing, horizontal line in fractions and radicands, etc. (see, among others, chapters 4 and 5).

Auxiliary parentheses can be repeated indefinitely without any room for confusion, since the closing signs occur in the reverse order to the corresponding opening signs.

3.2. Vertical delimiters

"Vertical delimiters" are graphical resources that define two or more expressions written one below the other. This is the case of the braces used to enclose the equations of a system and the formulas that define a "function." in pieces.

For example, in a system of equations. In the following case, the graphic resource is a key:

System of equations

$$\begin{cases} x + y + z = 1 \\ 2x - 3y - 4z = -2 \\ 4x + 2z = 0 \end{cases}$$

In this other case the graphic resource is a background of another color:

System of equations

$$\begin{cases} + + = 1 \\ 2 \ddot{y} 3 \ddot{y} 4 = \ddot{y} 2 \\ 4 + 2 = 0 \end{cases}$$

Regardless of the graphic resource used, the following will be transcribed in Braille:

1st) Blank line.

2nd) Vertical grouping indicator, which includes:

- Vertical delimiter sign:

#p (3456-1234) (system of equations) and #t (3456-2345) (function in pieces)



- Number of expressions to be delimited, as an ordinary number, # (3456 + braille sign of the 1st series).
- Sign 44 (1456-1456) (system of equations) and 84 (1256-1456) (piecewise function).
- If applicable, title or expression defining the structure.

3.º) The expressions to be delimited, on independent lines.

4th) Blank line, which will be omitted if it coincides with the end of a page.

The two previous examples will be written in braille in the same way:

```
#p#c44 {system of equations
x+y+z=#a
#bx-#cy-#dz=-#b
#dx+##bz=#j
```

All of these elements, including blank lines, must be contained in the same page. If there is no room for the last one, it will be deleted.

Here is an example with a piecewise function:⁶

$$f(x) = \begin{cases} 3x & \text{si } x < 0 \\ 6x & \text{si } x > 0 \end{cases}$$

```
#t#b84 f2x|=
#cx if x9#j #fx if
xo#j
```

The *less than* (<) and *greater than* (>) signs can be found in the chapter

6. *Arithmetic and algebraic relationships*. In Braille they are represented as 9 and 0, respectively.

⁶ See functions in chapter 9. *Analysis and calculation*.



In summary, the delimiters for system of equations and piecewise functions are:

Braille sign	Braille dots	Meaning
#p 44	3456-1234 and 1456-1456	system of equations
#t 84	3456-2345 and 1256-1456	piecewise function

3.3. Two-dimensional structures (matrices, determinants and tables)

A "two-dimensional structure" is any spatial arrangement of numerical quantities or individualizable mathematical expressions of any kind in rows and columns. Each such quantity or expression, regardless of its size and complexity, is called an "element" in one of these structures.

The most characteristic two-dimensional structures are considered to be "matrices" (enclosed in curved parentheses), "determinants" (between simple vertical bars) and "general tables", in which rows and columns can be distinguished, enclosed or not in a frame, with or without a grid, graphically affected by a certain background, with a rectangular or triangular appearance, etc.

The braille representation of a two-dimensional structure (matrix, determinant, general table or chart) includes:

1st) Blank line.

2.º) The "structure indicator", consisting of:

- The start sign corresponding to the two-dimensional structure:

• Curved parentheses (usually used for arrays): #s (3456-234).

• Simple vertical bars (usually used for determinants): #l (3456-123).

Any other representation (general tables): #v (3456-1236).

In this case, a Transcription Note must be included, in which the will describe the graphic resource that determines the table in the original (for example: framed table, with a background of a certain color, etc.)

- Number of rows (ordinary writing: number sign, character in (high position)).



- Multiplication sign (“cross”: $\times$ (236)).⁷
- Number of columns (ordinary writing: number sign, character in “high position”).
- Signs 54 (156-1456) for matrices, %4 (456-1456) for determinants and #4 (3456-1456) for tables.
- If applicable, title or expression defining the structure.

In summary, the delimiters for matrix, determinant and system are:

Braille sign	Braille dots	Meaning
#s 54 3456-234 and 156-1456 #l %4		matrix
3456-123 and 456-1456 #v #4 3456-1236		determinant
and 3456-1456		board

3.º) The braille representation of the two-dimensional structure may be made in two ways:

- **Explicit two-dimensional representation**, in which the following will be respected: as far as possible, the spatial distribution of the elements; in particular, the correspondence in columns (section 3.3.1).
- **Semi-linear representation**, facilitating student tasks especially when taking notes, where spatial correspondence in columns has a secondary value (section 3.3.2).

When editing and transcribing Braille texts, it is recommended to include both representations, separated by a blank line.

The first, the explicit one, is considered the most convenient and should always be included. For didactic reasons, and for the reader's knowledge, the second, semi-linear form should also be included, at least in the first examples of each work or chapter.

3.3.1. *Explicit two-dimensional representation*

Below, for each structure, are the opening (left) and closing (right) signs corresponding to each of the rows:

⁷ See chapter 5. *Arithmetic and algebraic operators*.

**Structure enclosed in parentheses (array):**

Row	Braille opening sign (left)	Points braille	Braille sign closing (right)	Points braille
Front row	S	0-234 5		156-0
Intermediate rows 1 2		0-123 %		456-0
Last row		0-126 ì		345-0

Structure enclosed between bars (determinant):

Row	Braille opening sign (left)	Points braille	Braille sign closing (right)	Points braille
Front row	%ÿ	0-456-0d %		456-0
Intermediate rows %ÿ %ÿ		0-456-0d %		456-0
Last row		0-456-0d %		456-0

Any other structure (tables in general):

Row	Braille opening sign (left)	Points braille	Braille sign closing (right)	Points braille
Front row	p	0-1234 4% 0-1236		1456-0
Middle rows lv		0-123	#	456-0
Last row				3456-0

Note that closures formed by points 456 require a blank space to be left after them and not a blank semi-enclosure, as is the general rule.

If it coincides with the end of the line, it will not be necessary to leave the space blank.



Example when the structure has vertical bars as delimiters:

$$\begin{vmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{vmatrix}$$

```
#l#c<#c%4 %
# a # b # c %
% # a j # b j # c j %
% # a , e # b # c %
```

Example when the structure has parentheses as delimiters:

$$\begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix}$$

```
#s#c<#c54 s#a
# b # c 5
l#a j # b j # c j %
2#a , e # b # c `
```

Example when the structure has any other structure as delimiters (in this case the structure is boxed):

1	23
10	20 30
1.5	2 3

```
#v#c<#c#4 p#a
# b # c 4
l#a j # b j # c j %
v#a , e # b # c #
```



In the following examples, ordinary parentheses will be used as delimiters, which are normally used for the representation of matrices.

If the matrix has only one row, it will be written as if it were the last one.

Example:

```
#s#a<#c54
2#a #b #c\
```

a) Two-dimensional structures with headers and footers in the rows and/or columns .

Column headers and footers will be written on separate, unspanned lines. by the structure delimiters (parentheses, slashes, brackets, etc.).

Row headers and footers will be written outside the row delimiters. structure and adjacent to them.

Example: 3×3 matrix from the previous examples, with column headers a, b, c and row headers 1, 2 and 3.

```
      a      b      c
1 ( 1      2      3 )
2 (10     20     30 )
3 (1,5    2      3 )
```

```
#s#c<#c54
      abc #as #a #b
#c5
#bl#aj #bj #cj%
#c 2 #a,e #b #ci
```

b) Columnarization. The elements of each column will be written in columns according to the most appropriate criterion according to the nature of said elements: with respect to the units figure, with respect to the number sign, etc.



Example: Tartaglia's Triangle.

```

      1 1
     1 2 1
    1 3 3 1
   1 4 6 4 1
  1 5 10 10 5 1

```

```

#s#e<#aa54
s #a #a 5
l #a #b #a %
l #a #c #c #a %
l #a #d #f #d #a %
2#a #e #aj #aj #e #a`

```

- c) **Separation between columns.** At least one column of a space will be respected in white between all elements of adjacent columns.
- d) **Ellipsis.** If ellipsis points appear in the structure to indicate an indefinite number of elements or other details will be represented in Braille by means of the sign ... (3-3-3), regardless of the direction represented (horizontal, vertical or oblique in any direction).

```

( 1  ...  3 )
( 10 ... 30 )
( 1,5 ... 3 )

```

```

#s#c<#c54
s#a ... #c5
l#aj ... #cj%
2#a,e ... #c`

```

- e) **Writing rows that exceed the length of the Braille display.** If the representation of the elements in a row of the two-dimensional structure exceeds the length of the Braille display:



- After the indicative expression of the structure and the header row of column, if applicable, the row headers will be written first—if any—, the opening delimiting sign and the complete columns of the structure that do not exceed the length of the Braille line, with the corresponding column headers or footers, if any.
- A blank line will be respected.
- The remaining columns of the structure will be written, with the corresponding column headers or footers - if any - the sign closing delimiter and row footers, if applicable.

Example:

$$\begin{pmatrix} 300 & 400 & 500 & 600 & 700 & 800 \\ a & b & c & d & e & f \end{pmatrix}$$

```
#s#b<#f54
s#cjj #djj #ejj #fjj
2 abcd
```

```
#gjj #hjj5 ef ì
```

If the number of rows in the structure exceeds the number of lines on the Braille page, the headers and initial rows of the structure will be written first - if necessary, split into two or more pages -, including the column headers - if they exist - on the last line of the Braille page, and then the writing will continue.
of the following rows, repeating the column headers, if any.

f) Successive horizontal structures. If the length of the Braille line allows, several complete structures related to each other can be written. The sign
The indicative of each of the structures will be written in the top row in columns with the opening sign of the structure.



$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \quad \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

#s#c<#c54 #s#c<#c54
 sa b c5 sa b c5 ld ef% ld ef%
 2g h il 2g h il

3.3.2. Semi-linearized Braille representation

It attempts to preserve two-dimensionality and a certain correspondence between columns of elements, allowing differentiation of the position in each line and the headers/footers of rows and columns. It dispenses with the space count required by a strict correspondence in column, which allows taking notes or making operations quickly.

The following sequence will be followed:

- 1st) Blank line.
- 2.º) The "structure indicative", which is formed with the structure sign, the number of rows, the multiplication sign, the number of columns, the closing sign, and the title or name, if applicable.
- 3.º) The first row starts with the structure indicator sign.
- 4.) Each element is separated by the "column separator" sign . (0-3).
 If an element is missing, nothing will be added in its place.
- 5.º) Each row, starting from the second, is written on a separate line, starting with the "row separator" sign ; (0-23).
- 6.) The last row concludes with the symmetrical "closure of the structure" sign from the "beginning" by vertical symmetry.
- 7th) Blank line.

The indicator and all rows of the semi-linearized structure must be in the same braille page.



Example:

$$\begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix}$$

#s#c<#c54

#s#a .#b .#c ;#aj .#bj .#cj ;#a,e .#b .#c54

a) Column headers. If they exist, they will be written as a first column.

line inside the matrix, starting with the structure indicator sign

and then the sign ; (23), separated by "column separator" signs . (0-3). The

rows will be represented as in the previous example,
where there were no column headers.

$$\begin{array}{ccc} a & b & c \\ \begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix} \end{array}$$

#s#c<#c54

#s;a .b .c ;#a .#b .#c ;#aj .#bj .#cj ;#a,e .#b .#c54

If there is no header in any of the columns, nothing will be put in its place.



$$\begin{array}{ccc} a & & c \\ \left(\begin{array}{ccc} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{array} \right) \end{array}$$

#s#c<#c54

#s;a . .c ;#a .#b .#c ;#aj .#bj .#cj ;#a,e .#b .#c54

b) Column footers. If they exist, they will be written on an additional line.

will begin with ;; (0-23-23). Each foot, starting with the second, will be preceded by the "column separator" sign . (0-3).

If there is no foot on any of the columns, nothing will be placed on it.

This additional line with the "column headers" will be closed with the structure closing symbol, keeping the 0-3 as the column separator.

$$\begin{array}{ccc} \left(\begin{array}{ccc} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{array} \right) \\ a & & c \end{array}$$

#s#c<#c54

#s#a .#b .#c ;#aj .#bj .#cj
;#a,e .#b .#c ;;a . .c54

c) Row headers. If they exist, they will be written at the beginning of the row, after the

"row separator sign", preceded by the distinctive sign @ (5-0)

and followed by a column separator sign . (0-3). This is done

achieves greater columnization. This pair of signs, @ (5-0), is will write to all rows even if there is no header in that row. In case of



If there is no header in a given row, since a row header indicator will be followed by a column separator, one will be suppressed.
of the two consecutive spaces that are generated.

$$\begin{matrix} A & \begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix} \\ C & \end{matrix}$$

#s#c<#c54
#s@

{a .#a .#b .#c ;@ .#aj .#bj .#cj ;@ {c .#a,e .#b .#c5

d) Row footers. If there is a row with a "footer", this will be written at the end of the row.

same after the indicator sign @ (5-0). If there is no "footer" in any row, the @ sign (point 5) must be written as the last character of the line.

$$\begin{matrix} & a & & c & \\ A & \begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix} & 1 \\ C & & & & 3 \\ & a & & c & \end{matrix}$$

#s#c<#c54
#s;a . .c ;@
{a .#a .#b .#c@ #a
;@ .#aj .#bj .#cj@ ;@
{c .#a,e . .@ #c ;;a . .c54

e) Writing rows that exceed the length of the braille line. If the

braille representation of a row exceeds the length of the braille display, it is

concludes with the "column separator" sign . (0-3) and continues in the next, respecting an indentation of two blank spaces followed by point 3. That is, the continuation of the line on the next line would begin with

. (0-0-3).



Example:

$$\begin{array}{ccccc}
 & a & & c & e \\
 r & \left(\begin{array}{ccccc} 1 & 2 & 3 & 5/2 & 14 \\ 10 & 20 & 30 & 25/12 & z \\ 1,5 & \dots & \dots & 545 & 4/3 \\ 545 & 623 & 161 & 488 & 487 \\ 115 & 644 & 331 & 622 & 758 \end{array} \right) & \begin{array}{l} x \\ t \\ y \end{array} \\
 & & b & & d
 \end{array}$$

#s#e<#e54

#s;a . .c . .e ;@

r .#a .#b .#c .#e; .#ad@ ;@ .#aj .#bj .#cj .#be; . .z@
x ;@ t .#a,e#ede . .#d: @

t ;@ .#ede .#fbc .#afa .#dhh . .#dhg@

and ;@

v .#aae .#fdd .#cca .#fbb .

.#geh@ ;; .b . .d .54

3.4. Delimitation of regions within a structure two-dimensional

The graphical differentiation of regions or groups of elements within a structure Two-dimensional is made with different graphic resources: text or background color differentiation, enclosure or linking by lines, vertical or horizontal delimiters...

However, all these resources seem to have the sole function of determining the region, there being no specific mathematical meaning in the graphic resource employee. The meaning of the region or the purpose of the sign is established by the context.



There is, however, one exception: tensors, which are usually represented as "arrays of arrays." In this case, the parentheses inside the general array have specific meaning.

In Braille, any of these forms will be represented by the signs and applying the rules listed below in sections 3.4.1 for the explicit two-dimensional representation, and 3.4.2 for the semi-linearized form.

Elements not marked by the same type of Braille signs can be considered as a distinct region, although they do not need to be marked.

3.4.1. Regions in two-dimensional structures represented in braille explicitly

Preceding the representation of the structure (and forming part of it), It includes a "Transcription Note" indicating the graphic value of the Braille differentiation, or of each of them, if there are several.

The delimitation of regions is carried out by enclosing said regions in "boxes" or "squares" formed by consecutive braille signs horizontally and vertically. In the We will call the explanations that follow "boxes".

To form these boxes, two collections of braille signs are considered, with independent meaning.

Primary or preferential

$\}::: ? $	0-35-25-25-25-26-0
$\% \%$	0-123-... -456-0
$e::: i$	0-123-... -456-0
	0-15-25-25-25-24-0

Secondary

$\},,, ? k$	0-35-2-2-2-26-0
$\{ k \{ e,,, i$	0-13-... -46-0
	0-13-... -46-0
	0-15-2-2-2-24-0

The "0" values of blank cells preceding the initial signs, or of the left, and that follow the ends, or on the right, are part of column separation spaces.



For the enclosure of typographically differentiated elements in Braille, will take into account the following issues:

- a) The spaces necessary to maintain the column of elements.
- b) The horizontal edges (top and bottom) are placed on independent lines and may not contain structural elements.
- c) The representation of the vertical edges of the box is omitted if they coincide with the global delimiters of the structure (parentheses, slashes, brackets, etc.).
- d) If in the same structure there are several regions with the same graphic differentiation but separated from each other, a single family of signs will be used in Braille.
- e) Elements not enclosed in a box can be considered as a distinct region, although they do not need to be enclosed in a separate box.
- f) For the places of intersection, where appropriate, of regions differentiated by the two families of Braille signs, signs resulting from the overlapping of both types of boundaries: r (1235) or w (2456).
- g) However, if horizontal edges of both differentiations coincide Braille, will be placed on independent lines.
- h) When the region to be highlighted is formed by an isolated element:
 - If the graphic differentiation is carried out by means of signs of the first family, they are preceded by the sign '}' (points 4-35).
 - If it corresponds to the second graphic differentiation, ^} (45-35).
- i) If the two differentiations coincide in the same element, the sign ^} (points 45-4-35).
- j) When the graphic differentiation consists of the linking by a line oblique (e.g., diagonally), each of the elements will be considered as "isolated element", applying the above.

The following examples illustrate some of these details.

If it were necessary to highlight the minor complementary to the element a31 (the one represented in red in the original):

$$\begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix}$$



In Braille it would be represented like this:

#s#c<#c54 }:::?:
s #al #b #c 5

l #aj l#bj #cj %
le:::~w
2 #a,e #b #c ì

If it were also necessary to indicate the symmetrical minor of the one corresponding to a31 (region consisting of elements 10, 20, 1.5 and 2, boxed in the original):

$$\begin{pmatrix} 1 & 2 & 3 \\ \boxed{10 \quad 20} & 30 \\ \boxed{1,5 \quad 2} & 3 \end{pmatrix}$$

#s#c<#c54 }:::?:
s #al #b #c 5

r,,,,,r,,? %
l #aj l#bj{ #cj %
le::w:::~w
l #a,e #b { #c ì
e,,,,,,,,,i

In the following example, elements 1, 20 and 3, which form one of the diagonals, have different typography:

$$\begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix}$$



In Braille its representation will be:

```
#s#c<#c54 s'}
#a #b #c 5
l #aj '}'#bj #cj%
2 #a,e #b '}'#c ì
```

In the following example, the graphic resource used to differentiate regions is a vertical delimiter, whose representation does not require the use of symbols of typographic variant.

$$\begin{pmatrix} 3 & 4 & | & 1 & 0 \\ 5 & 7 & | & 0 & 1 \end{pmatrix}$$

In Braille, its representation will be:

```
#s#b<#d54 s #c
#dl #a #j5
2 #e #gl #j #aì
```

3.4.2. Regions in two-dimensional structures represented in Braille in semi-linear form

If this representation is not preceded by the explicit two-dimensional representation, it must be preceded by a transcription note describing the correspondence, indicating which typographical device of the original is represented with the braille signs to indicate the various differentiations presented below.

In the semi-linear representation, the delimitation of regions (or groups of elements) is done row by row, using braille signs marking the first element belonging to the region and the last one, if applicable. The first one will be preceded by a sign distinctive and the last one will be followed by the corresponding distinctive sign. The following will be distinguished: different rows graphically differentiated: first row, intermediate rows and last row.

Two collections of Braille signs are considered, with independent meaning.



Preferred differentiation

Row	Braille start sign (left)	Points braille	Braille sign closing (right), in your case	Points braille
First row (top edge)	'?	0-4-26 '}	0-5-26 @}	4-35-0
Middle rows @? _?				5-35-0
Last row (bottom edge)		0-6-26 _}	0-4-26 _}	6-35-0
Single row	'?			6-35-0

Secondary differentiation

Row	Braille start sign (left)	Points braille	Braille sign closing (right), in your case	Points braille
First row (top edge)	^?	0-45-26 ^}		45-35-0
Middle rows {?		0-46-26 {}	0-56-26 "}	46-35-0
Last row (bottom edge)	"?			56-35-0
Single row	^?	0-45-26 "}		56-35-0

In its transcription into Braille, the following issues will be taken into account:

- Braille signs for differentiation of regions, both opening and closing closing, if applicable, have the value of column separator signs, so these are omitted.
- Closing signs are omitted if the differentiation extends to the end of the row, except in the case of a "single row" or if there are "row headers".
- If there are several regions with the same differentiation in the same structure graphic, the same family of signs will be used in Braille.
- When the region to be highlighted is formed by an isolated element:
 - If graphical differentiation is performed by primary differentiation, it is puts the sign '}' in front (4-35).



- If it is done by secondary differentiation, the sign is placed before it

$\wedge\}$ (45-35).

- If the two differentiations coincide in the same element, the first will be put first.

the sign $\wedge\}$ (45-4-35).

- e) When the graphic differentiation consists of linking or marking in a vertical or oblique sense (e.g., diagonally), each of the elements is
It will be considered as an "isolated element", applying the above-mentioned provisions.

The following examples illustrate some of these details.

In the following matrix it is necessary to highlight the minor complementary to the element a31 (the (represented in red in the original):

$$\begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix}$$

In Braille, the following will be represented:

```
#s#c<#c54
#s#a'?'#b .#c ;#aj'?
#bj .#cj ;#a,e .#b .#c54
```

If it were also necessary to indicate the symmetrical minor of the one corresponding to a31 (region consisting of elements 10, 20, 1.5 and 2, boxed in the original):

$$\begin{pmatrix} 1 & 2 & 3 \\ \boxed{10 & 20} & 30 \\ \boxed{1,5 & 2} & 3 \end{pmatrix}$$

```
#s#c<#c54
#s#a'?'#b .#c ;^?
#aj'?'#bj"}#cj ;^?#a,e .#b"}
#c54
```

In the following example, elements 1, 20 and 3, which form one of the diagonals, have different typography:

$$\begin{pmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 1,5 & 2 & 3 \end{pmatrix}$$

In Braille, its representation will be:

```
#s#c<#c54
#s'}#a .#b .#c ;#aj'}
  #bj .#cj ;#a,e .#b'}
  #c54
```



Chapter 4. Indexes and Marks

4.1. Positions

In visual characters, a sign can be affected by letters, numbers, expressions, brands, etc., in any of the positions shown in the graph following:

2 4 6

Z

1 3 5

Position 1: left subscript.

Position 2: left superscript.

Position 3: subscribed.

Position 4: superscript.

Position 5: right subscript.

Position 6: right superscript.

4.2. Indexes

4.2.1. General case

To indicate the position of the index, the following signs are used:

Braille written after the base sign they affect:

Visual sign	Braille sign	Braille dots	Meaning
ÿ ÿ _ ' 1		6-34	left subscript
ÿ ÿ		4-16	left superscript
ÿ ÿ	íí	34-34	subscribed index
ÿ ÿ	11	16-16	superscript index
ÿ ÿ í ÿ ÿ 1		34	right subscript (sub)
		16	right superscript (super)

The last two positions are the ones that appear most frequently.



Examples (using "z" as the base letter and "r" as the index):

Visual sign	Braille sign	Meaning
r_z	$z_{\dot{r}}$ lowercase	z with "r" in subscript on the left
r_z	$z'1r$ lowercase	z with superscript "r" on the left
z_r	lowercase $z\dot{r}$	z with an r subscript
r_z	$z11r$ lowercase	z with superscript "r"
zr	zircon	z with subscript "r"
zr	$z1r$	z with superscript "r"

If the index were made up of several terms or a mathematical expression, will be enclosed in auxiliary parentheses.

Examples (for writing numbers, see chapter 2. *Numbers and figures*, and for writing writing auxiliary parentheses (see chapter 3. *Delimiters*):

Visual sign	Braille sign	Meaning
z_{n-1}	$z\dot{r}\{n-1\}$	z with subscript "n-1"
$z_{i,j}$	$z1\{i,j\}$	z with superscript "i,j"
z_{i-r-1}	$z\dot{r}\{i-r-1\}$	z with subscript the difference between «i» sub «r» and 1
z_{i-r-1}	$z\dot{r}\{i-r-1\}$	z with subscript "i" affected by subscript "r-1"
z_{n-1}	$z'1\{n-1\}$	z with left superscript «n-1» $z1-$
$z^{-1/2}$	$\#a;$	z with superscript on the right -1/2

Similarly for any position.

In some cases, auxiliary parentheses aren't necessary, although their use doesn't alter their meaning. See the following example:



Visual sign	Braille sign	Meaning
zio	z íí#j z í?	z with subscript "i sub 0"
zio	íí#j}	z with subscript "i sub 0"

4.2.2. Numerical subscripts

In the representation of two-dimensional structures (see chapter 3) whose elements have numerical subscripts, these will be represented in abbreviated form using Braille elements of the fifth series (numbers in lower position) without position indicator or numerical sign.

If the subscript consists of two single-digit numbers separated by a comma, the comma may be omitted if there is no risk of confusion.

Visual sign	Braille sign
a ₁₁ a ₁₂ a ₂₁ a ₂₂	a,, a,; a:, a::
a _{1,1} a _{1,2} a _{2.1} a _{2.2}	a,, a,; a:, a::

4.3. Brands

4.3.1. Right superscript marks

Certain brands that are frequently used have a special representation in Braille.

The "prime," "second," and "third" marks in visual characters are represented by one, two, or three acute accents, respectively, in superscript positions. In Braille, each of these marks is written:

Visual sign	Braille sign	Braille dots	Meaning
ÿÿ		1256	premium
ÿÿÿ	8 88	1256-1256	second
ÿÿÿÿ 888 1256-1256	1256-1256		third



Other marks in this right superscript position are written below from the base sign using its specific sign followed by point 3. The main signs used as marks in the mathematical context are:

Visual sign	Braille sign	Braille dots	Meaning
$\overset{+}{y}$	$\overset{+}{.}$	235-3	superscript plus sign mark superscript
$\overset{-}{y}$	$\overset{-}{.}$	36-3	minus sign mark
$\overset{\circ}{y}$	$\overset{>}{.}$	356-3	superscript circle mark
$\overset{*}{y}$		256-3	superscript asterisk mark
$\overset{`}{y}$		26-3	grave accent mark
$\overset{'}{y}$	$\overset{*}{.} \overset{?}{.} 8.$	1256-3	apostrophe mark

When there is more than one brand, repeated or different, point 3 will be written only at the end.

Below are examples of how it is written, using the letter "z" as a base:

Visual sign	Braille sign	Meaning
z^+	$\overset{+}{z}$	z with a positive sign
z^-	$\overset{-}{z}$	z with a negative sign
z°	$\overset{>}{z}$	z with circle ⁸
z^*	$\overset{*}{z}$	z with asterisk
z^{+++}	$\overset{+++}{z}$	z with three positive signs
$z^{``}$	$\overset{??}{z}$	z with two grave accents
z^{*-}	$\overset{*}{z} \overset{-}{.}$	z with asterisk and minus

In the particular case of one of the special marks being repeated 4 or more times, it will be represented in Braille with the superscript sign followed by the number, the corresponding mark and a last character with the point 3.

⁸ When $^\circ$ is used to indicate *degrees* it is not followed by a period (v. 11.5. *Angle measurements*).



Example:

Visual sign	Braille sign	Meaning
z4*	z1#d*.	z with four asterisks
z****	z1#d*.	z with four asterisks

Any other brand not reflected in the table above will be represented after the base letter, preceded by the braille sign indicating the position in superscript to the right, ending with point 3.

zÿ z1#8. z with superscript infinity sign on the right⁹

4.3.2. Superscript marks

Certain superscript braille marks have a special representation. they write before the base letter ("z" or "Z" in the examples).

Visual sign	Braille sign	Meaning
z̄	'cZ	superstriped lowercase zeta
ẑ	'c'c{z	capital letter zeta with two horizontal strokes in superscript
z̃	@?z	wavy line over lowercase z
zÿ	'>z	circle over lowercase zeta
ẑ	":z	circumflex over lowercase zeta
z̃	"-z	inverse circumflex over lowercase zeta
ẑ	':z	arch over capital letter Z
ẑ	_':z	reverse arc on zeta

⁹ See chapter 7. *Set theory* for the sign of "infinity".



In the particular case of letters marked with one, two or three superscript dots, it is necessary to use the corresponding alphabetic prefix, even for lowercase Latin letters, as seen in the following examples:

Visual sign	Braille sign	Meaning
$\overset{\cdot}{R}$	'{r "z	capital R with a period
$\overset{\cdot\cdot}{r}$	Greek letter zeta lowercase with two dots "'@r lowercase r	
$\overset{\cdot\cdot\cdot}{r}$	with three dots	

If the mark affects a compound expression, this expression will be enclosed between the corresponding auxiliary parentheses.

Visual sign	Braille sign	Meaning
$\overline{AB}$	'c?{a{b} lines over A and B	
$\overline{z''}$	'c?z88} zeta second superstriped	

Any other mark not reflected in the previous cases will be represented after the base letter preceded by 16-16 and ending with point 3.

$\overset{++}{Z}$ z11++. zeta with two plus signs superscript

4.3.3. Trademarks in the subscriber

Certain brands in Braille have a special representation. They are written before the base letter.

Visual sign	Braille sign	Meaning
$\underset{\sim}{z}$	__?z	wavy line under zeta
$\underline{z}$	__-z	underlined z
$\underline{\underline{z}}$	__-_-z z with double underscore	



If the mark affects a compound expression, this expression will be enclosed between the corresponding auxiliary parentheses.

Visual sign	Braille sign	Meaning
<u>ab</u>	$_ - ? ab \}$ a and b underlined $_ - ? z88 \}$ zeta	
<u>z''</u>	second underlined	

Any other mark not reflected in the previous cases will be represented after the base letter, preceded by 34-34 and ending with point 3.



zi9. zeta with "less than" sign in subscript

4.3.4. Marks in subscript on the right or left or superscript on the left

When any of the marks listed in the previous sections appear in any of these positions, they will be written preceded by the Braille position indicator, corresponding to the base letter and followed by point 3:

Visual sign	Braille sign	Meaning
z+	$z \acute{+} .$	z with positive sign in subscript on the right
----z	$z'1\#d- .$	z with four negative signs in superscript position on the left z second
++z'' z88	$_ \acute{+} + .$	with two plus signs in subscript position on the left

4.4. Symbols affected by multiple indices and/or marks

4.4.1. Writing order

When a symbol or letter is affected by several indexes and/or marks, it must be follow the following order:

- 1) Subscribed marks that do not require a Braille position sign.
- 2) Superscript marks that do not require a Braille position sign.
- 3) Base symbol.
- 4) Mark that does not require a superscript position indicator on the right.



- 5) Marks and/or indexes in subscript on the left.
- 6) Marks and/or indexes in superscript on the left.
- 7) Brands and/or indices in subscription.
- 8) Marks and/or indexes in superscript.
- 9) Marks and/or indexes in subscript position on the right.
- 10) Marks and/or indexes in superscript position on the right.

Note that for writing indices of letters and numbers that require positional signs, the writing order determined in section 4.1 is followed.

Examples:

Visual sign	Braille sign	Meaning
$z_{\substack{3 \\ 4}}$	$z\acute{i}\#d1\#c$	z with subscript "4" and superscript "3"
$z_{i,j}^2$	$z\acute{i}\?i ,j\}1\#b$	z with subscript "i,j" and superscript "2"
z'_0	$z8\acute{i}\#j$	z prime with subscript "0"
z'^3	$z81\#c$	z prime with superscript "3"
$\overline{z}_0$	$'c?z\acute{i}\#j\}$	z with subscript "0" superlined
$\overline{(z'_0)}^2$	$'c2z8\acute{i}\#j\}1\#b$	z prime with subscript "0", in parentheses, superlined and with superscript "2"

If this expression does not contain parentheses, for its transcription into Braille it is will use auxiliary parentheses.

$\overline{z'_0}^2$ 'c?z8\acute{i}\#j\}1\#b z prime with subscript "0", superlined and with superscript "2"

4.4.2. Shifted indexes

Braille writing of the displaced indexes will be done by writing, before the corresponding position indicator, the sign " (56) for subscripts and the sign ^ (45) for superscripts.



The index closest to the base sign will always be written first.

Visual sign	Braille sign	Braille dots	Meaning
ÿÿ ÿ	$\wedge 1$	45-16	shifted superscripts
ÿ ^ÿ	"Yo	56-34	shifted subscripts

In these two examples the "s" is displaced:

Visual sign	Braille sign	Meaning
Tr _s	{tír^1s T subscript «r», shifted superscript «s» {t1r"ís T superscript	
Tr _s	«r», shifted subscript «s»	

Note in the previous case that, following the rule of writing "indexes" "shifted" the superscript that affects the base letter is written first, instead of the subscript. If the subscript is not shifted, the order would be the reverse, as you can see below (see section 4.3).

T_s^r {tís1r



Chapter 5. Arithmetic and algebraic operators

5.1. Elementary arithmetic operations

Visual sign	Braille sign	Braille dots	Meaning
+	+	235	add
=	=	2356	equal to
-	-	36	subtract
X	<	236	multiply (cross)
.	⠨⠼	6-0d	multiply (point)
/	*	256	division with slash (exclusively when indicating division)
—	*	256	division with horizontal line
: ÷ ·/ @,		5-2	division with two points (in any of the variants in which they appear)
⠨⠼, ⠨⠼, ⠨⠼ †, ⠨⠼	" .	56-3	generic operator (usually used to represent an operation between elements of a set)

Examples:

Visual sign	Braille sign	Meaning
6+2 #f+ #b 6-2		six plus two
#f-#b 6x2 #f<#b		six minus two
6·2 #f_ #b 6⠨⠼2		six by two
#f".#b 3b #c@b		six by two
		six operated with two
		three b



Visual sign	Braille sign	Meaning
$3a+5x$	$\#c@a+\#ex$	three or more five x
$a+bc$	$a+bc \ x \cdot y$	a plus b minus c
x_y	$x\ddot{y}y \ x'' \cdot y$	x by y
$3:4$	$\#c@, \#d$	x operated with y
$\frac{3}{4}$	$\#c^*\#d$	three divided by four (with two points)
$\frac{3}{4}$	$\#c^*$	three game by four, with horizontal bar ¹⁰
$—$	a^*c	a split by c, with horizontal or oblique bar
$—$	a^*cx	a divided by c, with a horizontal bar, multiplied by x
$—$	$a^*?c_x\}$	a divided with a horizontal bar by the product of c by x (with multiplication point)
$\frac{+}{—}$	$?a+b\}*c$	a plus b divided by c with a horizontal bar
$\frac{+ \ -}{+}$	$?a+b^*c\}*?d+e\}$	fraction whose numerator is the sum of a and the quotient from b to c, and whose denominator is d plus e
$+ / a+b^*c$		a plus b divided by c, with horizontal or oblique bar
$\frac{\frac{a+b}{c+d}}{x+y}$	$??a+b\}*?c+d\}\}*?x+y\}$	fraction whose numerator is the fraction of numerator a plus b and denominator c plus dy whose denominator is x plus y
$\frac{3}{5} \cdot \frac{2}{7} = \frac{6}{35}$	$\#c? _ \#b= =\#f:?$	three-fifths times two-sevenths equals six thirty-fifths

¹⁰ For the abbreviated representation, see chapter 2.



5.2. Powers and roots

5.2.1. Powers

Since, from a graphical point of view, the exponent of a power is a superscript, it will be written after the base preceded by the braille indicator

corresponding 1 (points 16) —see 4.2—.

Examples:

Visual sign	Braille sign	Meaning
x^2	$x1\#b$	x squared
	$x1n$	x raised to an
x^{-1}	$x1\#a$	x raised to -1
$x^{\frac{1}{2}}$	$x1\#a; x1?$	x raised to $\frac{1}{2}$
x^{a+b}	$a+b\}$	x raised to the sum of a+b
$x^{-(a+b)}$	$x1-2a+b\} x1n1\#b$	x raised to minus (a+b)
x^{2n}		x squared n

Visual sign	Braille sign
$7^3 - 2^2 + + 1 \#g$	$x1\#c-\#bx1\#b+x+\#a$
$3a + \frac{1}{2}^{23}$	$\#c@a+\#a; x1\#by1\#c$
$\frac{2+1}{2-1}$	$?x1\#b+\#a\}*?x1\#b-\#a\}$
$\frac{4}{3^2}$	$\#d*?\#c@a1\#b\}$
$+$	$x1a+b$



5.2.2. Roots

$\sqrt{\quad}$

1246 ... 156

root sign

The root index is placed between the two braille elements that make up the sign, and the radicand is written after it.

Example:

$\sqrt[3]{8}$

6#c5#h Cube root of eight

In the case of the square root, the index (2) is omitted by analogy with writing in ink.

Example:

$\sqrt{8}$

65#h Square root of eight

When the radicand (graphically: mathematical expression covered by the line) horizontal of the root sign) is a compound expression, it is enclosed in auxiliary parentheses unless the removal of these would not cause confusion, as shown in the last example.

Examples:

Visual sign	Braille sign	Meaning
$\sqrt{\quad}$	65x	square root of x
$\sqrt[n]{a+b}$	6n5?a+b} $\sqrt{\quad}$	nth root of a plus b
$\sqrt[n]{a+b}$	6n5a+b	to the nth root of a b is added
$\sqrt[n-1]{\frac{a}{b}}$	6n-#a5?a*b}	root of index n-1 of a divided by b
$\sqrt{x^2 + y^2}$	65?x1#b+y1#b}	square root of the sum of x squared plus and squared
$\sqrt[3]{2^2 - 9}$	6#c5?#c@a1#ba}+9	cube root of difference of three to al square minus ay, outside the root, plus 9
$\sqrt{\sqrt{16}}$	6565#af	square root of the square root of 16



5.3. Combinatorics

Visual sign	Braille sign	Braille dots	Meaning
$n!$	$n^{\wedge}. n^{\wedge}.^{\wedge}.$	1345-45-3	factorial of n
$n!!$	1345-45-3-45-3	double factorial of n	
$\binom{n}{r}$	$\{2n:r\}$	46-126-1345-25-1235-345	binomial coefficient n over r

As you can see in the following examples, permutations, variations and combinations, with or without repetition, will be written as they appear in the original, but without the need to respect the index shift.

$$C_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{combinations without repetition}$$

$$\{c\acute{in}1k=\{2n:k\}=n^{\wedge}.*?k^{\wedge}.2n-k\}^{\wedge}.$$

$$CR_n^k = \binom{n+k-1}{k} \quad \text{combinations with repetition}$$

$$\{c\{r\acute{in}1k=\{2n+k-\#a:k\}$$

$$= = - \frac{!}{(\ddot{y})!} \quad \text{variations without repetition}$$

$$\{v\acute{in}1k=n1_k=n^{\wedge}.*2n-k\}^{\wedge}.$$

$$= \quad \text{variations with repetition}$$

$$\{v\{r\acute{in}1k=n1k$$

$$, = (\ddot{y} 1)(\ddot{y} 2) \dots (\ddot{y} + 1) \quad \text{variations without repetition}$$

$$\{v\acute{í}?n,k\}=n2n-\#a\grave{2}n-\#b\grave{2}n-k+\#a\grave{2}$$

Visual sign	Braille sign	Braille dots	Meaning
$\pm$		235-25-36	more or less
$\ddot{y}$		36-25-235	less or more
%		456-356	percentage
‰		456-356-356	so much per thousand
$ \dots $		456-0d-...-456-0d	absolute value; module (vertical bars)

6 ± 2 #f+:-#b

21% #ba%>

```
| | = 1 %'a% =#a absolute value of alpha equal to one
```

`|x| = 1 % x% = #a absolute value of x equals one`

5.5.1. Summation or summation

45-234-...-156
 ^S 5
 y

The limits of the sum, if they appear at different levels, are placed between the Greek letter uppercase sigma ^s(45-234) and the sign5(156), separated by the sign:(25).

$$\ddot{y}_{4=1}^2 \quad \text{sum of } j=1 \text{ up to } 4 \text{ of } j^2$$
$$^s j = \#a : \#d5j1 \#b$$
$$\sum_{j=1}^4 j^2$$
$$^s\#a_9=j_9=\#d_5j_1\#b$$



In this example it is not necessary to write the opening and closing parentheses signs in Braille, since the limits of the sum are already delimited by the summation sign. as opening and 156 as closing.

5.5.2. Producer

$$\pi^p 5 \quad 45-1234-\dots-156$$

Product boundaries, if they appear at different levels, are placed between the letter Greek capital pi $\pi^p(45-1234)$ and the sign 5(156), separated by the sign: (25).

Examples:

$\prod_{j=1}^n S_j$ product, from $j=1$ to n , of the S_j

$$\pi^p_{j=\#a:n} 5 \{yesj\}$$

$\prod_{j=1}^n S_j$ producer for 1 less than or equal to j less than or equal to n of S_j

$$\pi^p_{\#a9=j9=n} 5 \{yesj\}$$

In this example, it is not necessary to write the opening and closing parentheses in Braille since the limits of the sum are already delimited by the summation sign, as opening, and 156 as closing.

5.5.3. Co-producer

$$\prod^7 5 \quad 45-12456-\dots-156$$

The coproduct boundaries, if they appear at different levels, are placed between the sign $\wedge^7(45-12456)$ and sign 5 (156), separated by the sign: (25).

Example:

$$X = \prod_{j \in J} X_j$$

$$\{x = \wedge^7 j 2, \{j 5 \{x i j\}$$

The most common functions on sets of numbers are included below. represented by symbols/textual notations.

In Braille, they will be represented in the same way as in visual characters. These representations will end with point 3.



Visual sign	Braille sign	Meaning
max	max.	maximum
min	min.	minimum
mcd	gcd.	greatest common divisor
mcm	lcm.	lowest common multiple
inf	inf.	tiny
sup	sup.	supreme

The original delimiters (parentheses, brackets or braces) as well as separators (comma or semicolon) will be used in ink.

Examples:

gcd(28,34) gcd.2#bh ,#cdì

max[28;36] maxá#bh ;#cfú

5.6. Operators on two-dimensional structures

For writing in Braille, they will be represented as follows:

- Blank line.
- Next, a first line in which the headers of the structures to be operated are written, separated by the operator signs. If the Braille line, is interrupted by one of these signs and is repeated on the line next, respecting an indentation of two blank spaces.
- Blank line.
- The operating structures are represented in the manner indicated in section 3 of chapter 3. The operation sign will be placed in the position that best suits the adjustment.
- Blank line.

Example:

$$A + B = \begin{pmatrix} -2 & 4 \\ 2 & 6 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ -3 & 5 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -1 & 11 \\ -1 & -1 \end{pmatrix}$$

#s{a+{b54=

s-#b #d5 s #b #a5 s #j #e5

l #b #f% + l-#c #e% = l-#a #aa%

2 #j -#a\ 2-#a #j\ 2-#a -#a\



Chapter 6. Arithmetic and algebraic relationships

6.1. Relationships

Visual sign	Braille sign	Braille dots	Meaning
=	=	2356	equal to
≈	'= =?	4-2356	approximately equal to
≈	"= "":=	2356-26-0	questioned whether it is equal to
≈		56-2356	equal by definition
≈	56-25-2356		corresponds to
≈	==	2356-2356	identical to; congruent with
	%≈	456-0d	divides to
≈	%*	456-256	proportional to
≈	"."	56-23	proportion; equality of ratios
≈	@,	5-2	ratio
≈	@?. 5-26-3		tilde operator; similar to; difference between; proportional to
≈	@=. 5-2356-3		almost equal to; asymptotically equal to
≈	{:k 46-25-13 {:		geometric progression
÷		46-25	arithmetic progression

Examples:

$p \approx m \cdot v$ $p = m \cdot v$ p equal by definition a is by v $A \approx B$ $\{a @ = \{b A$

almost equal to B

$4 \approx 3 \cdot 8$ $\#d @, \#c"; \#h @, \#f$ 4 is to 3 as 8 is to 6



Other relationships:

Visual sign	Braille sign	Braille dots	Meaning
$< \nless \lessdot \lessgtr$	9	246	less than
$> \ngtr \gtrdot \gtrless$	or	135	greater than
$\ddot{y}$	99	246-246	much smaller than
$\ddot{y}$	00	135-135	much greater than
$\ddot{y} \ddot{y} \ddot{y} 9=$		246-2356	less than or equal to
$\ddot{y} \ddot{y} \ddot{y} 0= @9 0, @99$		135-2356	greater than or equal to
$\ddot{y} \ddot{y}$		5-246	prior to
$\ddot{y}\ddot{y}$		135-2	after
$\ddot{y}$	5-246-246 00,	135-135-2	long before
$\ddot{y}$	@9= 5-246-2356 0,=		much later than
$\ddot{y} \ddot{y}$	135-2-2356		equal to or prior to
$\ddot{y}\ddot{y}$			after or equal to

6.2. Denied relationships

The braille representation of the negation of a given relationship has as its first character points 45, followed by the representation of the relationship that is denied.

For example:

Visual sign	Braille sign	Braille dots	Meaning
$\ddot{y}$	$\wedge=$	45-2356	different from
$\ddot{y} \ddot{y} \wedge 09$	45-135-246		neither greater nor less than
$\ddot{y}$	$\wedge @ = .$	45-5-2356-3	is not asymptotically equal to



Visual sign	Braille sign	Braille dots	Meaning
ÿ	^@?. 45-5-26-3	^%ÿ	crossed-out up-down tilde
ÿ	45-456-0d ^9		is not a divisor of
ÿ		45-246	not less than
ÿ	^o	45-135	no greater than
ÿ	^9= 45-246-2356	^o=	is not less than or equal to
ÿ	45-135-2356 ^@9	45-5-246	is not greater than or equal to
ÿ	^o, 45-135-2		is not prior to
ÿ			is not later than



Chapter 7. Set Theory

7.1. Basic concepts

Visual sign	Braille sign	Braille dots	Meaning
=	=	2356	equal to
ÿ	==	2356-2356	identical to
{	@l	5-123	open set wrench
}	%,	456-2	close set key
(	@2	5-126	open class key
)	ì,	345-2	close class key
ÿ	@á	5-12356	open class bracket
ÿ	ú, ,	23456-2	close class bracket
,		0-2	element-separating comma
	%ÿ	456-0d	such that
:	@,	5-2	
;	;	0-23	
ÿ	—,	6-2	
ÿ	%j	456-245	empty set
	%u	456-136	set or universal class
ÿ	2,	126-2	is an element of; belongs to
ÿ	@ì	5-345	comprises; has as an element
ÿ	^2, 45-126-2 ^@ì		is not an element of; does not belong to
ÿ	45-5-345 2.		does not understand; does not have how element
ÿ		126-3	Included; subset



Visual sign	Braille sign	Braille dots	Meaning
$\ddot{y}$	2; _ Ì	126-23	included or equal to
$\ddot{y}$	"Ì ^2.	6-345	Includes; has as a subset
$\ddot{y}$		56-345	includes or is equal to
$\ddot{y}$	45-126-3 ^ _ Ì	45-6-345	not included in
$\ddot{y}$	^2; 45-126-23 ^"Ì		does not include
$\ddot{y}$	45-56-345		not included nor equal to
$\ddot{y}$			does not include or equal to

7.2. Operations between sets

Visual sign	Braille sign	Braille dots	Meaning
$\ddot{y}$	% Ì	456-345	union
$\ddot{y}$	%5	456-156	intersection
$\ddot{y}$	\ Ì 5 123456-345-...-156		union of a family of sets
$\ddot{y}$	\5 5 123456-156-...-156 @. ^d {<		intersection of a family of sets
$\ddot{y}$	"c{a	5-3	difference of sets
$\ddot{y}$		45-145	symmetric difference of sets
X		46-236	Cartesian product
	56-14-46-1 {a8	46-1-1256	complementary to A
TO'	'c{a 4-14-46-1		complementary to A
			complementary to A



Examples:

Let $= \{2, 4, 6, 8, 10\}$ and $= \{ : \ddot{y} \ddot{y}, < 7 \}$

$\{let \{a=@l\#b, \#d, \#f, \#h, \#aj\%,$
and $\{b=@lx@, x2, \%n, x9\#g\%,$

B is the set of elements x such that x belongs to the set of numbers natural and x is less than 7.

$\ddot{y} = \{2, 4, 6\} \{a\%5\{b=@l\#b, \#d, \#f\%,$

$\ddot{y} \quad \ddot{y} \quad \backslash j2, \{i5\{aíj$

Union for j belonging to a set of indices I of the family $A_{sub j}$.

For the union and intersection of several sets, the rules seen for the summation and production (see chapter 5) will be applied.

7.3. Relationships between the elements of a set

Visual sign	Braille sign	Braille dots	Meaning
$\ddot{y}$	$x"ry$	1346-56-1235-13456	x related by the relation $\ddot{y}$ with y
$\ddot{y}-$	$\wedge"r$	45-56-1235-1, @9 o,	denial of the relationship $\ddot{y}$
$\ddot{y}$	@9=	6-2	quotient set
$\ddot{y}$		5-246	precedes; prior to
$\ddot{y}$		135-2	follows; after
$\ddot{y}$	5-246-2356 o,=	135-2-2356	precedes or equals to
$\ddot{y}$	$\wedge@9$	45-5-246 $\wedge$ o, 45-135-2	follows or equals to
$\ddot{y}$	$\wedge@9=$	45-5-246-2356	is not prior to
$\ddot{y}$	$\wedge$ o,=	45-135-2-2356	is not later than
$\ddot{y}$			is neither prior nor equal to
$\ddot{y}$			is neither later nor equal to



7.4. Cardinals

Visual sign	Braille sign	Braille dots	Meaning
Card	{card. 46-14-1-1235-145-3		cardinal of a
#	set	3456-13	cardinal of a set
$\aleph$	#k	3456-1256	infinite
#8 8 $\aleph$		6-1256	aleph
	#p	3456-1234	power of a set



Chapter 8. Logic

8.1. Quantifiers

Visual sign	Braille sign	Braille dots	Meaning
$\forall$	$\{.$	46-3	universal quantifier: for everything
$\exists$	$\{?$	46-26	existential quantifier: there exists at least one element
$\exists!$	$\{;$	46-23	unitary quantifier: there is only one element
$\nexists$	$\wedge\{?$	45-46-26	does not exist

8.2. Logic of propositions

Visual sign	Braille sign	Braille dots	Meaning
$\forall$	$\%0=$	456-2356	tautology
$\nexists$	$\wedge\%0=$	45-456-2356	no tautology
$\forall$	$\%S$	456-234	true proposition
$\forall$	$\%2$	456-126	false proposition
$\forall$		456-24	disjunction (or)
$\forall$	$\%Y0\%?$	456-26	conjunction (and)
$\forall$	$\backslash i\ 5\ 123456-24\dots15611$		disjunction of a family of propositions
$\forall$	$\backslash ?\ 5\ 123456-26\dots15611$		conjunction of a family of propositions
$\underline{\forall}$	$\%and$	456-2346	exclusive disjunction

¹¹ If indices appear, the system indicated in chapters 5 and 7 for summation will be followed.
 producer...: lower index, points 25, upper index.



Visual sign	Braille sign	Braille dots	Meaning
$\neg$	$_.$	6-312	logical negation
$\sim$	$_.$	6-312	logical negation
$\ddot{y} 9:o \ddot{y} :o \ddot{y} 9:$		246-25-135	double implication "if and only if"
$_1 'í$		25-135	direct implication; implies
		246-25	reciprocal implication; because
$\ddot{y}$		0-6-16-0	therefore
$\ddot{y}$		0-4-34-0	since

Examples:

$\ddot{y} , \neg(\ddot{y}) \ddot{y} \neg \ddot{y} \neg \{.p , q\} .2p\%?$

$q\ddot{y} 9:o _ .p\%i _ .q$

$\ddot{y} : \ddot{y} \sim = \ddot{y}$

$\wedge \{?p@ , q\%? _ .p=\% =$

¹² Whatever the visual representation of the logical negation, in Braille it will be represented by the sign 6-3.



Chapter 9. Analysis and calculation

9.1. Generalities

This section contains examples of how general function notations are written. They use the symbols used in basic writing, as well as some specific symbols for representing functions, which are listed below.

For the representation of functions in Braille the individual letter that is used will be used. corresponds to the original ink without using the typographic variant symbol (f, g, h...).

Visual sign	Braille sign	Braille dots	Meaning
$\longrightarrow$	$::$	25-25-2	correspondence or application between sets
$\longleftarrow$	$@::$	5-25-25	reverse correspondence
$\longleftrightarrow$	$@::$	5-25-25-2	bijective application

When the function arrow is affected by an index, this is written in braille between the two elements 25 that the arrow contains:

$\dot{y}$

$:F:$

Visual sign	Braille sign	Braille dots	Meaning
$:$	$@$	5-2, as the two points are represented on other occasions (division, ratio, such that...)	colon (when used in functions)
f^{-1}	$f1-\#a$ $fo.$	124-16-36-3456-1 124-346-3 (abbreviated form)	inverse function of f
$\ddot{y} ==$		2356-2356	identical to
$\ddot{y} _;$		6-23	composition of functions



Writing ordered pairs

In Braille, as in visual characters, ordered pairs are written in parentheses separated by commas.

Examples:

Visual sign	Braille sign	Braille dots	Meaning
$(1, 2)$	$2x\acute{a}, x\grave{b}$	126-1346-34-3456-1-0-2-1346-34-3456-12-345	ordered pair x_1, x_2
$\left(\frac{1}{2}, \frac{3}{2}\right)$	$2\#a; \#c; \grave{}$	126-3456-1-23-0-2-3456-14-23-0-345	ordered pair one half, three half

For the representation of common symbols in Function Theory, the rules established in section 5.5 are followed.

For example:

Visual sign	Braille sign	Braille dots	Meaning
Sun	{sun. 46-145-135-134-3 {or.		Domain
Or	46-135-1235-3		Original
Img Im	{img. {im.	46-24-134-1245-3 46-24-134-3	Image
conc	conc. 14-135-1345-14-3		concave
conv	conv. 14-135-1345-1236-3 convex		
grow	crec. 14-1235-15-14-3		growing
dec	Dec. 145-15-14-3		decreasing
mant	maintain 134-1-1345-2345-3 decimal or		mantissa



Visual sign	Braille sign	Braille dots	Meaning
arg	arg. 1-1235-1245-3		argument
rot	rot. 1235-135-2345-3		rotational or rotor
div	div. 145-24-1236-3		divergence
grad	grad. 1245-1235-1-145-3	gradient	
Re	{re. 46-1235-15-3		Royal part
Im	{im. 46-24-134-3		Imaginary part
Rec	{rec. 46-1235-15-14-3		Route

Any other symbol/notation will follow the regulations in chapter 5.

Examples:

Visual sign	Braille sign	Meaning
$f: A \rightarrow B$	$f@, \{a::, \{b$	application f of A to B
$f: A \rightarrow B$	$\{a:f, \{b$	application f of A to B
$B \xrightarrow{f^{-1}} A$	$\{b:f1- \#a: \{a$	reverse application of f , from B to A (f to the -1 of B to A)
$B \xrightarrow{f^{-1}} A$	$\{b:fó::, \{a$	reverse application of f , from B to A (short form)
$f: A \rightarrow B$	$\{a@::, \{b$	bijective application of A to B
$f(x) = y$	$f2x\lceil=yf==\#j$	f of x is equal to y
$f \equiv 0$		f is identical to zero
$f \circ g$	$f2x\lceil=g2x\lceil$	composition of functions



Piecewise function

$$f(x) = \begin{cases} 3x & \text{si } x < 0 \\ 6x & \text{si } x > 0 \end{cases}$$

#t#b84 f2xì=
#cx if x9#j #fx if
xo#j

See *Chapter 3*, where this form of representation was established.

Intervals

Common parentheses or brackets are used just like in characters visuals.

Visual sign	Braille sign	Meaning
], [	úa ,bá	open interval with endpoints a and b (,) 2a ,bì open
interval with endpoints a and b	] áa ,bú	closed interval with endpoints
[, a and b		
[, [	áa ,ba	interval closed on the left and open on the right
[,)	áa ,bì	interval closed on the left and open on the right
] ,]	úa ,bu	interval open on the left and closed on the right
(,]	2a ,bu	interval open on the left and closed on the right



9.2. Limits

Visual sign	Braille sign	Braille dots	Meaning
	lim. :,	123-24-134-313	limit
$\dot{y}$		25-2	tends to
$\ddot{y}$	%.	456-3	tends to decrease to
$\ddot{y}$	%to	456-1	tends to grow to
$\overline{lim}$	'clim. 4-14-123-24-134-3	upper limit	
	limsup.	123-24-134-234-136-1234-3	upper limit
—	_lim. 6-36-123-24-134-3	lower limit	
	liminf.	123-24-134-24-1345-126-3	lower limit

If the "limit" has subscribed information in visual characters, in Braille this is will write between the "limit" sign and the character 156.

Visual sign	Braille sign	Meaning
$\dot{y}$	$x:,c \dot{y}$	x tends to ac
$\ddot{y} x:,\#8$		x tends to infinity
$\lim_{\dot{y}}$	lim.x:,,c5	limit as x tends to ac
$\lim_{\ddot{y}}$	lim.x%ac5	limit as x tends to increase ac
$\lim_{\dot{y}0\dot{y}}$	lim.x:,,#j-.5	limit as x tends to 0 from the left
$\lim_{\ddot{y}}$	lim.x%.c5	limit as x tends to decrease ac

¹³ See what was agreed in section 5.5 regarding symbols and notations.



Visual sign	Braille sign	Meaning
$\lim_{x \rightarrow 0^+}$	$\lim.x:,\#j+.5$	limit as x tends to 0 from the right

To simplify the braille representation in developments in which a character is repeated expression of the limit operator, the form l5 (123-156) can be used , indicating between these two signs the value to which the variable tends. This must be explained in a *Braille Transcription Note*.

Examples:

$$\lim_{x \rightarrow c} (f(x) + g(x)) = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$$

$$\lim.x:,\#5f2x\grave{\imath}+g2x\grave{\imath}=\lim.x:,\#5f2x\grave{\imath}+\lim.x:,\#5g2x\grave{\imath}$$

If the expression $\lim.x:,\#5$ is replaced by the shorthand form lc5, its transcription would be:

$$lc5f2x\grave{\imath}+g2x\grave{\imath}=lc5f2x\grave{\imath}+lc5g2x\grave{\imath}$$

In the example below,

$$\lim_{y \rightarrow \frac{1}{2}} \frac{2 + 3}{-5} = \frac{y}{y} = \lim_{y \rightarrow \frac{1}{2}} \frac{\frac{2}{2} 3 + \frac{3}{2}}{\frac{1}{2} - \frac{5}{2}} = \lim_{y \rightarrow \frac{1}{2}} \frac{1 + \frac{3}{2}}{\frac{1}{2} - \frac{5}{2}} = \frac{1}{0} = \frac{y}{y}$$

In Braille , $\lim.x:,\#85$ has been replaced by the abbreviated form l#85; its transcription would be:

$$l\#85?x1\#b+\#cx\}^*?x-\#e\}=\#8*\#8=\#l\#85?$$

$$x1\#b*x1\#b+\#cx*x1\#b\}^* *?x*x1\#b-\#e*x1\#b\}=\#l\#85?\#a+\#c*x\}^*?\#a*x-\#e*x1\#b\}=\#a*\#j=\#8$$

$$\#a*\#j=\#8$$



9.3. Derivatives and differentials

To represent increments, Δ (uppercase delta) will be used.

For the representation of differentials, the lowercase **d** will be used without using any typographic variant symbol.

Visual sign	Braille sign	Braille dots	Meaning
Δ		45-145	increase in...
d	$\wedge dd$	145	differential of...
∂	$\%d$	456-145	partial derivative

Derivative of a function

Visual sign	Braille sign	Meaning
$\frac{df}{dx}$	$\{df\}^*\{dx\}$	derivative of « f » with respect to « x »
$d_x f$	$d^*\{dx\}f$	derived, with respect to « x », from « f »
$\frac{d^2 f}{dx^2}$	$\{d^2 f\}^*\{dx^2\}$	derivative of « f » of « x »

If a function f is derivable at all points of its domain, the derivative function, which is usually denoted as f' , is defined.

For example:

If $f(x) = 2$, then $f'(x) = 0$

If $f(x) = x^2$, then $f'(x) = 2x$



Derivative of order n

Visual sign	Braille sign	Meaning
—	$?d1nf\}^*?dx1n\}$	n th derivative of « f » with respect to « x »
—	$d1n^*?dx1n\}f$	n th derivative, with respect to « x », of « f »

As an alternative option, which abbreviates the braille representation of the derivatives, you can use the following sequence:

- 1st) Sign formed by the points 1456.
- 2nd) Order of the derivative if it is greater than one.
- 3rd) Derivation variable.
- 4th) Sign formed by the points 156.
- 5th) Function to be derived.

The abbreviated notation will be explained in an explanatory note.

Visual sign	Braille sign	Meaning
—	$4x5f$	derivative with respect to " x " of " f "
—	$4x5f$	derivative with respect to " x " of " f "
—	$4nx5f$	n th derivative with respect to « x », of « f »
—	$4nx5f$	n th derivative with respect to « x », of « f »

Partial derivative

— Partial derivative of « f » with respect to « x ».

$?%df\}^*?%dx\}$



— Partial derivative, with respect to «x», of «f».

$\%d^*\%dx\}f$

nth partial derivative

— nth partial derivative of "f" with respect to "x" n times

$\%d1nf\}^*\%dx1n\}$

— nth partial derivative of "f" with respect to "x", n times.

$\%d1n^*\%dx1n\}f$

$\frac{2}{\text{—}}$ Second partial derivative with respect to "x" and "y".

$\%d1\#b^*\%dx\%dy\}$

$\frac{2}{\text{—}}$ Second partial derivative of "f" with respect to "x" and "y".

$\%d1\#b@f\}^*\%dx\%dy\}$

$\frac{+}{\text{—}}$ Partial derivative of order "m+n", with respect to "x" m times and with respect to
of "and" n times

$\%d1?m+n\}^*\%dx1m\%dy1n\}$

$\frac{+}{\text{—}}$ Partial derivative of order "m+n" of "f", with respect to "x" m times
and with respect to "y" n times

$\%d1?m+n\}f\}^*\%dx1m\%dy1n\}$

As an alternative option, which abbreviates the braille representation of partial derivatives, the following sequence can be used:

1st) Sign formed by the points 56-1456.

2nd) Order of the partial derivative.



3rd) Variable or variables with respect to which the derivative is performed.

4th) Sign formed by the points 156.

5th) Function to be derived.

Visual sign	Braille sign	Meaning
—	"4x5f	partial derivative with respect to "x" of "f"
—	"4nx5f	nth partial derivative of "f" with respect to "x" n times
—	"4nx5f	nth partial derivative with respect to "x" n times, of "f"
² —	"4#bxy5f	second partial derivative of "f" with respect to "x" and "y"
² —	"4#bxy5f	second partial derivative of "f" with respect to "x" and "y"
⁺ —	"4m+nxmyn5f	partial derivative of order "m+n" of "f", with respect to "x" m times and with respect to "y" n times
⁺ —	"4m+nxmyn5f	partial derivative of order "m+n", with respect to "x" m times and with respect to "y" n times

Example:

$$\frac{\partial^2}{\partial x \partial y} (f(x, y)) = \left(\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) \right)$$

$$\frac{\partial^2}{\partial x \partial y} (f(x, y)) = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial x \partial y}$$

Shorthand notation:

$$\frac{\partial^2}{\partial x \partial y} (f(x, y)) = \frac{\partial^2 f}{\partial x \partial y}$$



Nabla, Laplacian, d'Alambertian

Visual sign	Braille sign	Braille dots	Meaning
∇		4-12456	nabla operator
Δ	'7%<	456-236	Laplacian operator
$\square$	%and	456-13456	Alambertian operator

9.4. Integrals

Visual sign	Braille sign	Braille dots	Meaning
$\int$	& 5	12346-...-156	integral sign
$\int$	& 5	12346-156	indefinite integral
$\iint$	&& 5 12346-12346-...-156		double integral
$\iiint$	&&& 5	12346-12346-12346-...-156	triple integral
$\oint$	&>5	12346-356-156	contour integral
$\oint_C$	&>{c5 12346-356-46-14-156		contour integral along curve C
$\int_{\sigma}$	&.'s 5 12346-3-4-234-...-156		line integral
$\otimes$	@;	5-23	convolution product

The representation in visual characters of "definite integral" is characterized because the integral sign is accompanied by a subscript and a superscript, both at the right of the sign. This information will be written in Braille between the two characters that form the integral sign, writing first the subscript already

then the superscript, both separated by the braille character : (25).



Visual sign	Braille sign	Meaning
$\int_a^b$	&a:b 5 definite integral between a and b	
$\overline{\int_a^b}$	'c&a:b 5 superior definite integral between a and b	
$\underline{\int_a^b}$	_-&a:b 5 definite lower integral between a and b	

Examples:

$$\int x^2 dx$$

$$\int_1^4 x^2 dx$$

$$\int_1^4 x^2 dx = \left[\frac{1}{3} x^3 \right]_1^4 = 21$$

$$\int_1^4 x^2 dx = \frac{1}{3} x^3 \Big|_1^4 = 21$$



Chapter 10. Classical Functions

10.1. Sequences, progressions and series

10.1.1. Successions

Visual sign	Braille sign	Braille dots	Meaning
(s_n)	2sín 126-234-34-1345-345		sequence of general terms s sub n
$\{s_n\}$	@lsín%,	5-123-234-34-1345-456-2	sequence of general terms s sub n
S_n	without	234-34-1345	sequence of general terms s sub n

Examples:

$$() = (1, 3, \dots, 2 + 1, \dots)$$

$$2sín\grave{=}\grave{=}2\#a ,\#c ,\dots ,\#bn+\#a ,\dots\grave{=}$$

$$\{ \} = \{ 0\grave{y}9, 0\grave{y}99, \dots, 1\grave{y}10 \quad \frac{1}{\dots}, \dots \}$$

$$@l\{pík\%,=@l\#j,i ,\#j,ii ,\dots , \\ \#a-\#a*?\#aj1k\} ,\dots\%,$$

Note that in the case of ellipses it is not necessary to leave a space in white in front.

In the transcription of documents, the representation used by the author will be respected.

Example:

$$\lim_{n \rightarrow \infty} \quad \text{Limit of } s_n \text{ as } n \text{ tends to infinity}$$

$$\lim_{n \rightarrow \infty} s_n$$

Using the abbreviation for *lim.* (see 9.2. Limits):

$$l\#85s_n$$



10.1.2. Progressions

Visual sign	Braille sign	Braille dots	Meaning
$\div$	$\{:$	46-25	arithmetic progression
$\ddot{y}$	$\{:k$	46-25-13	geometric progression

10.1.3. Series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \lim_{k \rightarrow \infty} \sum_{n=1}^k \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{n^2} &= \lim_{k \rightarrow \infty} \sum_{n=1}^k \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \\ &= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \end{aligned}$$

10.2. Functions represented by literal symbols/notations

In Braille, they will be represented in the same way as in visual characters. These representations will end with point 3.

Following the established criteria, these notations can be replaced by others that are frequently used in any of the countries in which this code will apply.

Some of them are listed below.

10.2.1. Exponential and logarithmic functions

Visual sign	Braille sign	Meaning
exp	exp.	exponential (base e)
log _b x	log. _b x	logarithm to base b of x
logx	log.x	decimal logarithm of x
Ln x ln x	ln.x	natural or natural logarithm of x



Visual sign	Braille sign	Meaning
alog x antilog x	alog.x antilog.x	antilogarithm of x
aln x	aln.x	natural or natural antilogarithm of x
cologx colog.x	cologarithm of x	

Decimal logarithms with negative characteristic: to write in Braille the characteristic of a decimal logarithm when it is negative, the following will be used: Braille characters of the third series (see *Introduction*) by prefixing them with the sign numeric.

Examples:

$$\log 0,2 = \bar{1},30103$$

The decimal logarithm of 0.2 has characteristic -1 and mantissa 30103

log.#j,b=#u,cjajc

-28,345 Decimal logarithm of characteristic -28 and mantissa 345

#go,cde

10.2.2. Trigonometric (circular) functions

Visual sign	Braille sign	Other representations	Meaning
without sen	without. sen.		breast
cos	cos.		cosine
so tg	so. tg.		tangent
cot cotg	cot. cotg.		cotangent



Visual sign	Braille sign	Others representations	Meaning
sec	sec.		drying
csc harvest	csc. cosec.		cosecant
arcsin arcsen	arcsin. arcsen.	asin. asen.	arc sine
arccos	arccos. acos. arccosine		
arctan arctg	arctan. arctg.	tie. atg.	arc tangent
arccot arccotg	arccot. arccotg.	note. acotg.	arc cotangent
arcsec	arcsec. asec. secant arc		
arccsc arccosec	arccsc. arccosec.	acsc. acosec.	cosecant arc

Visual sign	Braille sign	Meaning
version verses	version. verses.	verseno
coversin coversen	coversin. coversen.	coversene
semiversion semiverses	semiversion. semiverses.	semiverse



Visual sign	Braille sign	Meaning
semi-covers semi-covered	semi-version. semi-covered.	semi-verse

10.2.3. Hyperbolic functions

Visual sign	Braille sign	Others representations	Meaning
sinh senh	sinh. senh.	sh.	hyperbolic sine
cosh	cosh.	ch.	cosine hyperbolic
tanh tgh	tanh. tgh.	th.	hyperbolic tangent
coth ctgh	coth. ctgh.	cth.	hyperbolic cotangent
dry	sech.	sch.	drying hyperbolic
csch harvest	csch. harvest.	csch.	cosecant hyperbolic
argsinh argsenh	argsinh. argsenh.	ash.	hyperbolic sine argument
argcosh	argcosh. ach.		argument cosine hyperbolic
argtanh argtgh	argtanh. argtgh.	ath.	hyperbolic tangent argument



Visual sign	Braille sign	Others representations	Meaning
argcoth argcotgh	argcoth. argcotgh.	acth.	hyperbolic cotangent argument
argsech	argsech. asch.		secant argument hyperbolic
argcsch argcosech	argcsch. argcosech.	acsch.	cosecant argument hyperbolic

10.2.4. Other functions, parameters and characteristics represented by symbols

Visual sign	Braille sign	Braille dots	Meaning
int	int. 24-1345-23	45-3 whole part	
dec	Dec. 145-15-14-3		decimal part
sgn	sign. 234-1245-13	45-3 sign	
abs	abs. 1-12-234-3		absolute value
Re	{re. 46-1235-15-3		real part of a complex number
Im	{im. 46-24-134-3		imaginary part of a complex number
rg	rg. 1235-1345-3		range
Adj	{adj. 46-1-145-245-3		attached

If a function or literal symbol does not appear in this chapter, the same will be followed. rule that in section 10.2. That is, point 3 later, with transcription note indicating its meaning.



Chapter 11. Geometry and vectors

Note: The letters shown are not part of the defined symbols. They are generic and may be different depending on the context.

11.1. Geometric elements

Visual sign	Braille sign	Braille dots	Meaning
$\ddot{y} @:,r$		5-25-2-1235	straight line r
$\overleftrightarrow{PQ}$	@:,{p{q}	5-25-2-26-46- 1234-46-12345-35	line determined by points P and Q
$\ddot{y} :,z$		25-2-1356	right ray z
$\ddot{y} @:z$		5-25-1356	left ray z
$\overrightarrow{OA}$	:{,o{a}	25-2-26-46-135- 46-1-35	ray of origin O passing through point A
$\overline{AB}$	'c?{a{b}	4-14-26-46-1-46- 12-35	segment AB
$\ddot{y} ^:{a$		45-25-46-1	angle A
$\widehat{aob}$	^:{?aob}	45-25-26-1-135- 12-35	angle aob
$\ddot{y}\ddot{y}\ddot{y} \% -$		456-36	right angle
$\ddot{y} \{5$		46-156	positive facing angle
$\ddot{y} \{\grave{}$		46-345	negative facing angle
$\widehat{s}$	' :s	4-25-234	arch s
$abc \grave{?} abc\}$		26-345-26-1-12- 14-35	arc corresponding to angle abc



11.2. Vectors

Visual sign	Braille sign	Braille dots	Meaning
$\vec{v}$	$\dot{:,v}$	25-2-1236	vector v
$-\vec{v}$	$@:v$	5-25-1236	opposite vector of vector v
$ \vec{v} $	$\% \ddot{:}, v \% \ddot{}$	456-0d-25-2-1236-456-0d	module of the vector v
$ \vec{v} $	$\% :, v \% $	456-123-25-2-1236-456-123	norm of the vector v
$\overrightarrow{\alpha}$	$\underline{5}$	6-156	positive axial vector
$\overleftarrow{\alpha}$	$\text{'}\alpha$	4-345	negative axial vector

Note: Where there is no room for confusion based on the context, the right arrow may be omitted, as shown in one of the dot product examples.

Note: as already stated in previous chapters, "0d" indicates that the sign The Braille to be written in this location must not have dot 1, dot 2, or dot 3 (a blank space on the right). If this condition is not met, a blank space must be left between the "closing bar" sign and the following sign.

Examples:

$|\overrightarrow{AB}|$ Module of vector AB $\% \dot{:, ?\{a\{b\}}\%$

$||\overrightarrow{AB}||$ Norm of the vector AB $\% |:, ?\{a\{b\}}\% |$

$\overrightarrow{\alpha}$ Positive axial vector alpha $\underline{5}$ 'a

$\overleftarrow{\alpha}$ Negative axial vector alpha ' l'a

$[\overrightarrow{AB}]$ Free vector AB $\acute{a}:, ?\{a\{b\}}\acute{u}$



11.3. Geometric relationships

Visual sign	Braille sign	Braille Dots	Meaning
$\ddot{y}$	{9	46-246	It is secant with
$\ddot{y}$	%	456-123	is parallel to
#	% =	456-123-2356	is parallel and equal to
$\ddot{y}$	#.	3456-3	perpendicular to orthogonal to
$\ddot{y}$	"?;	56-26-23	homologous to/similar to
$\sim$	@?.	5-26-3	equivalent to/similar to 14
$\overline{\wedge}$	%6	456-1246	projectivity
$\overline{\overline{\wedge}}$	%7	456-12456	perspectivity
$\ddot{y}$	@?=	5-26-2356	congruent with

Examples:

$\overline{\overline{\wedge}} \parallel \overline{MN}$ The line $\ddot{y}$ is parallel to the segment with ends M and N

@:,|%'c?{m{n}

$\overline{AB} \perp \overline{OX}$ The line determined by points A and B is perpendicular to the ray of origin O that contains point X

@:,{a{b}#.:,{o{x}

Note: Regarding the respective negations of each of these relations, the rules seen for the relations dealt with in Chapter 6 apply. That is, the signs of "negation" of the existence of a relation (or "negative relations") in Braille have as their first character the sign formed by the dots 45.

¹⁴ Also in chapter 6.



For example:

Visual sign	Braille sign	Braille Dots	Meaning
$\ddot{y}$	$\wedge 0 $	45-456-123	not parallel
$\ddot{y}_\perp$	$\wedge \# .$	45-3456-3	not perpendicular

11.4. Operations between geometric elements

Visual sign	Braille sign	Braille dots	Meaning
$\vec{v} \cdot \vec{w}$	$:,V_ : ,W$	25-2-1236-6-0- 25-2-2456	scalar product of vector v by vector w
$\langle \vec{v}, \vec{w} \rangle$	$@k:,v : ,w\{,$	5-13-25-2-1236- 0-2-25-2-2456- 46-2	dot product of the vector v by the vector w

This is an example where, if it is clear from the context that v and w are vectors, the arrows can be removed:

$$\vec{v} \cdot \vec{w} \quad V \cdot W$$

$$\langle \vec{v}, \vec{w} \rangle \quad @kv ,w\{,$$

Visual sign	Braille sign	Braille dots	Meaning
$\times$	$'<$	4-236	vector product
$\times \ddot{y}$	$'<$	4-236	vector product
$\ddot{y}$	$" ,$	56-2	vector product
$\ddot{y}$	$9+$	246-235	direct sum
$\ddot{y}$	$9<$	246-236	tensor product
$\hat{\oplus}$	$\%9+$	456-246-235	orthogonal sum
$+\ddot{y}$	$'+$	4-235	vector addition



Visual sign	Braille sign	Braille dots	Meaning
$\vec{y}$	' _	4-36	vector subtraction

Note: when there is no room for confusion, these last two signs are usually replaced by the common signs of addition and subtraction.

Examples:

$$\vec{v} \times \vec{w}$$

$$\vec{v} + \vec{w} \quad : , v' < : , w \quad : , v' + : , w \quad : , v + : , w$$

11.5. Angle measurements

In accordance with the provisions for the notation of units in general (see *Appendix 1*) the representation of the units of measurement of angles does not include the point 3 at the end.

In the particular case of the sexagesimal and centesimal degrees, as well as the respective minutes and seconds, there will never be blank space between the numerical coefficient and the unit.

Visual sign	Braille sign	Braille dots	Meaning
	rad	1235-1-145	radian
$\vec{y}^\circ$	>	356	sexagesimal degrees
$\vec{y}\vec{y}$	8	1256	sexagesimal minutes
$\vec{y}\vec{y}\vec{y}$	88	1256-1256	sexagesimal seconds
$\vec{y}$	1g	16-1245	hundredths
$\vec{y}\vec{y}$	18	16-1256	hundredths of a minute
$\vec{y}\vec{y}\vec{y}$	188	16-1256-1256	hundredths of a second



Examples:

90° ninety sexagesimal degrees

#ij> (3456-24-245-356)

37° 22' 49" thirty-seven degrees twenty-two minutes forty-nine seconds

#cg>#bb8#di88

(3456-14-1245-356-3456-12-12-1256-3456-145-24-1256-1256)

11.6. Polygons and curves

In the case of polygons, auxiliary parentheses are not used to enclose the letters that designate the vertices, since once the figure is identified, the number of vertices.

According to the previous observation, the representation in visual characters of the polygons will always consist of the drawing of the corresponding figure in position superscribed over the letters that designate the vertices.

The Braille representation will be carried out by writing the letters of the vertices after the corresponding sign that characterizes the figure.

Visual sign	Braille sign	Braille dots	Meaning
$\triangle$ <i>abc</i>	_úabc 6-23456-1-12-14	triangle abc	
$\ddot{y}$	%<	456-236	right triangle
$\ddot{y}$	%y	456-13456	square
$\ddot{y}$	&y	12346-13456	rectangle
$\ddot{y}$	&o	12346-135	polygon
O		246-135	circle
$\ddot{y}$	9o ?}	26-35	geometric curve



Chapter 12. Statistics and Probability

Because many expressions related to this topic consist solely of Latin or Greek letters—upper or lower case—the transcription generally follows the respective visual character notation. According to this criterion, the list

The list of notations that we will present below is not exhaustive and the user of this code may introduce those that he considers necessary, adjusting to these guidelines and others already established in previous chapters, which are reflected in the following examples.

Likewise, what has already been established for the representation of the symbols/notations (see section 10.2. *Functions represented by symbols/literal notations*).

When a symbol/notation consists of a set of two or more characters, will always end with dot 3. If a symbol/notation is given by a single character, point 3 will not be added.

For this purpose, a letter and its alphabetical prefix are considered to constitute a single character. Thus, the letter "K" or the letter alpha are considered to be made up of a single character.

Visual sign	Braille sign	Braille dots	Meaning
.	'cX	4-14-1346	average
	'm	4-134	population average
ÿ ÿ	@kx{, 5-13-1346-46-2		population average
	{h	46-125	harmonic mean
	{g	46-1245	geometric mean
	{mo.	46-134-135-3	fashion
	{me.	46-134-15-3	median
	{r	46-1235	statistical range
	{i{q{r. {q	46-24-46-12345-46-1235-3	interquartile range
		46-12345	quartile



Visual sign	Braille sign	Braille dots	Meaning
	{p	46-1234	percentile
2	's1#b.	4-234-16-3456-12-3	population variance
	{var. 46-1236-1-1235-3		variance
	'sr	4-234	standard deviation
		1235	correlation coefficient
	{and	46-15	hope or expected value
	{q{v.	46-12345-46-1236-3	quasi-variance
	{c{v. 46-14-46-1236-3		's coefficient of
		4-234	population covariance
	variation {cov. 46-14-135-1236-3		
	covariance {d{m. 46-145-46-134-3		
		4-134	central moment
		1234	probability
	mean deviation 'mp		confidence interval
		15	mistake
		46-1345	normal distribution
ÿ		45-1245	Gamma distribution
		46-124	Fisher distribution
		46-136	uniform distribution
2	{i{c. 46-24-46-14-3 e {n ^g {f {u ^&1#b.	45-12346-16-3456-12-3	chi-square distribution



Visual sign	Braille sign	Braille dots	Meaning
	{p	46-1234	Poisson distribution
	{be.	46-12-15-3	Bernoulli distribution
		46-12	binomial distribution
		2345	Student's t distribution
		13	kurtosis
ÿ		45-2456	sample space
		124	frequency
	{btk ^wf {f	46-124	cumulative frequency

If necessary for the use of any computer program, it may be introduced some distinguishing character before the sign. For example, é (2346).



Chapter 13. Financial Mathematics

Because many expressions related to this topic consist solely of Latin or Greek letters—upper or lower case—the transcription generally follows the respective visual character notation. According to this criterion, the list

The list of notations that we will present below is not exhaustive and the user of this code may introduce those that he considers necessary, adjusting to these guidelines and others already established in previous chapters, which are reflected in the following examples.

Likewise, what has already been established for the representation of the symbols/notations (see section 10.2. *Functions represented by symbols/literal notations*).

When a symbol/notation consists of a set of two or more characters, will always end with dot 3. If a symbol/notation is given by a single character, point 3 will not be added.

For this purpose, a letter and its alphabetical prefix are considered to constitute a single character. Thus, the letter "K" or the letter alpha are considered to be made up of a single character.

Visual sign	Braille sign	Braille dots	Meaning
C	{c	46-14	capital
.	{i	46-24	interests
TAE	{t{a{e. 46-2345-46-1-46-15-3		annual equivalent rate or effective annual rate
to		1	annuity
.		24	interest rate
M	ai	46-134	future amount or value
D	{m	146-145	discount
d		145	discount type
TO		46-1	amortization
n	{dd {an	1345	periods

Visual sign	Braille sign	Braille dots	Meaning
GOES	{v{a. 46-1236-46-1-3		present value

If necessary for the use of any computer program, it may be introduced some distinguishing character before the sign. For example, é (2346).



Chapter 14. Adaptation and editing criteria

14.1. Aspects to take into account when writing texts mathematicians

Mathematical expressions will generally be written without spaces (blanks) except in those cases where a blank box forms the part of the sign, as can be seen in the preceding chapters (examples: delimiters — comma, semicolon, colon—, therefore...).

Examples:

$$3+4=7 \text{ #c+#d=#g}$$

$$1, 2, 3, 4 \text{ #a ,#b ,#c ,#d}$$

To avoid possible confusion, it is recommended not to use Braille shorthand in texts on exact or natural sciences.

When a mathematical formula appears included in a text, two characters will be left. spaces before and two spaces after the formula. If the end of the formula coincides with the end of the line, it will not be necessary to leave any space at the beginning of the next line.

Examples:

Sea $(\cdot) = \log + \log$ the expression we have left is...

{let $\log.2\{m<\{n\}=\log.\{m+\log.\{n$ be

The expression that remains with us is...

Knowing that $\log = \log$ the expression we have left is...

{knowing that $\log.\{m*\{n\}= =\log.\{m-\log.\{n$
the expression

What we have left is...



Sometimes, as an editorial resource, mathematical expressions are placed on separate lines:

Knowing that

$$\log = \log \ddot{y} \log$$

The expression we are left with is...

{knowing that

$\log.\{m*\{n\}=\log.\{m-\log.\{n$ the expression
we have left is...

14.2. Partitioning mathematical expressions

In principle, no expressions or structures should be split. When a formula is too long, it may be necessary to split it. To do this, keep the following rules in mind:

1. Generally, expressions will be split by relational operator symbols and then by arithmetic operators. The symbol will be repeated at the beginning of the next line.
2. An exception to this rule are expressions of sets defined by extension or by comprehension, successions, etc., which can be broken after a punctuation mark (such as "comma", "semicolon", "colon"), which will not be repeated in the next line.

Examples:

$$= \{ , , , , , , \dots , , \}$$

$$\{a=@|a ,b ,c ,d ,e ,f ,g ,h , \\ i , j , k , l , m\%,$$

«C is the set of x such that x belongs to the set of natural numbers, 6 divides ax and x is less than or equal to 12».

$$= \{ \ddot{y} \ddot{y} , 6| , \ddot{y} 12\}$$

$$\{c=@|x_,x2,\%n ,\#f\% x ,x9= \\ 9=\#ab\%,$$



3. There are a series of expressions, unary operators represented by symbols/notations (limits, logarithms, etc.), which should not be broken. Always go along with your argument. If the argument doesn't fit, you have to use the rules general partitioning by signs. The argument is given below and is will leave if necessary.

Examples:

$$\lim_{y \rightarrow y} (2^3 + 15) = \lim_{y \rightarrow y} (2^3) = +\ddot{y}$$

$$\lim_{x \rightarrow x} (+852 \# \text{bx1} \# \text{c} + \# \text{aexl}) = \\ = \lim_{x \rightarrow x} (+852 \# \text{bx1} \# \text{cl}) = +\#8$$

$$\lim_{y \rightarrow y} (2^3 - 15^3 (+8 \ddot{y} 5 + 12)) = \lim_{y \rightarrow y} (2^3 - 15^3 + 12) = \ddot{y}\ddot{y}$$

$$\lim_{x \rightarrow x} (-852 \# \text{bx1} \# \text{c} - \# \text{aey1} \# \text{c} + \# \text{hx} - \\ - \# \text{ey} + \# \text{abl}) = \lim_{x \rightarrow x} (- \\ \#852 \# \text{bx1} \# \text{c} - \# \text{aey1} \# \text{c} + \# \text{abl}) = -\#8$$

Equal signs do not have to appear only at the beginning of a line, as long as the breaks occur according to the general hyphenation rules:

$$\lim_{x \rightarrow x} (-852 \# \text{bx1} \# \text{c} - \# \text{aey1} \# \text{c} + \# \text{hx} - \\ - \# \text{ey} + \# \text{abl}) = \lim_{x \rightarrow x} (-852 \# \text{bx1} \# \text{c} - \\ - \# \text{aey1} \# \text{c} + \# \text{abl}) = -\#8$$

4. If the partition has to be performed in an implicit multiplication (for example, if an expression should be transcribed in which the sign does not appear to multiply in the original), you must add the multiplication symbol in braille.

Examples:

If the expression $39 \ddot{y} 147 + 44$

$$\# \text{cix} - \# \text{adgy} + \# \text{ddz}$$



should be cut due to lack of space immediately after 147, a multiplication sign would have to be added before the cut and repeated on the line following:

39 ÿ 147 ×

× + 44

#cix-#adg<
<y+#ddz

5. For the representation of quantities or structures that do not fit on a line and cannot be split by delimiters or operation signs (i.e., there is no other partition option), the math hyphen \ (123456) will be used at the end of the line, it will be repeated at the beginning of the next line and will continue without number sign, in the case of a quantity.

Example:

Approximation of the number with thirty-eight decimal places:

3.14159265358979323846264338327950288419

#c,adaeibfecehigicbchdfbfdcchcb\
\giejbhhdai

The most common operator signs by which an expression can be split are:

= = > or < 9	equal to
ÿ o= greater	greater than
than or equal	less than
to ÿ 9= less than or equal to	

ÿ much larger than

ÿ 99 much smaller than

ÿ '= approximately equal to

+ +	further
-----	---------



$\ddot{y} - \times <$	less
$\div * \pm$	multiply (blades)
$+;-$	divided by

more or less

$\ddot{y} \%?$ logical conjunction

$\ddot{y} \%i$ logical disjunction

$\ddot{y} \%5$ intersection of sets

$\ddot{y} \%l$ union of sets

$\ddot{y} 2.$ included in (sets)

$\ddot{y} _l$ includes (sets)

14.3. Other aspects to consider

1. It is recommended (primarily to editors) that science texts include a list of the symbols used and their meaning.
2. In chapter 3 you can find out what the function and use of the auxiliary parentheses. It is advisable to make them known when they appear first time in a text, since it is a resource specific to the system braille.
3. Numbers or expressions crossed out in mathematical operations (not valid for relational negations).

In the event that crossed-out operands must be included due to simplification processes or resolution methods, the situation will be represented in Braille. through successive equalities, as shown in the following examples.

Optionally, the complete operation may be transcribed, including the crossed-out operands preceded, without blank spaces, by the

sign 7 (12456). To group crossed-out terms, if they were not grouped by other delimiters (parentheses, brackets, braces, etc.), they must be used auxiliary parentheses. The use of the strikethrough sign will be noted in the timely "Braille transcription note".



Examples:

- a) Transcription without including the strikethrough marks, with the step-by-step simplification that, in general, does not appear in the ink version of the same situation:

$$\frac{2 - (\ddot{y}2)(\ddot{y}1)32}{2 - (+1)(\ddot{y}1)} = \frac{(\ddot{y}2)(\ddot{y}1)(\ddot{y}1)2}{(+1)(\ddot{y}1)} = \frac{(\ddot{y}2)(\ddot{y}1)2}{(+1)}$$

?#bx2x-#b\2x-#a\1#c}* *?#bx2x+#a\2x-
#a\}= =?2x-#b\2x-#a\2x-#a\1#b}* *

*?2x+#a\2x-#a\}= =?2x-#b\2x-
#a\1#b}* *2x+#a\

- b) Transcription without including the strikethrough marks, only with simplification final, which in ink is not written as an equality:

$$\frac{2 - (\ddot{y}2)(\ddot{y}1)32}{2 - (+1)(\ddot{y}1)} = \frac{(\ddot{y}2)(\ddot{y}1)2}{(+1)}$$

?#bx2x-#b\2x-#a\1#c}* *?#bx2x+#a\2x-
#a\}= =
=?2x-#b\2x-#a\1#b}* *?2x+#a\

- c) Full transcript including the "strikethrough" sign:

$$\frac{2 - (\ddot{y}2)(\ddot{y}1)3\text{---}}{2 - (+1)(\ddot{y}1)\text{---}} = \frac{(\ddot{y}2)(\ddot{y}1)2}{(+1)}$$

?7?#bx}2x-#b\7?2x-#a\1#c}}*
*?7?#bx}2x+#a\72x-#a\}= =?2x-#b\2x-
#a\1#b}*2x+#a\



14.4. Highlighting and coloring elements and expressions

When it comes to highlighting a text or formula in some way, the adaptation approach will be tactile rather than visual.

If a mathematical expression appears boxed in a book to be transcribed, the reproduction of boxes that include Braille characters should be avoided as far as possible, as this can cause confusion.

In this sense, a formula preceded by a text in words can be highlighted writing it with a blank line before and after it, which will simulate a box. You can also define indents or centering to help make it stand out.

The use of colors serves to reinforce the importance of a text, a mathematical expression, or part of one.

The colors are not part of a mathematical expression, but rather contribute to highlighting what the author considers important to be noted.

If the colors were omitted, the mathematical expression would not be modified, although the reference to those colors in the Braille text would have to be adapted.

The author or editor who adds colors does so thinking of one of the qualities of the sense of sight, which allows us to visualize the texts at a glance.

Highlighted. That is, colors are added assuming that the reader can appreciate them, discern them, and improve the understanding of a text through that perception that can be verified with a single glance. This type of perception cannot be verified by a blind reader.

Below are a series of illustrative examples of how to proceed in different cases that are not subject to regulations.

When it is necessary to differentiate letters in different colors, the signs will be used collected in chapter 1. *Literal characters*, section 1.3. *Typographic variants from the Latin alphabet*.

These are:

- 1st typographic variant prefix in lowercase: Braille sign _ (6).
- Prefix of 1st typographic variant in capital letters: braille sign " (56).
- 2nd typographic variant prefix in lowercase: braille sign ó (346).
- 2nd typographic variant prefix in uppercase: Braille sign 3 (146).



Example:

The colors mentioned are those different from the default one.

$X+y+z=H$

$\{x+y+oz=3h$

The capital letter X is black, the lowercase letter y is red, the lowercase z is green, and the Capital H is green.

If it is necessary to indicate that one of the digits of a number is written with a special typography, the third series of the Braille alphabet can be used, which is made up of the first ten characters with which the numbers are formed plus the points - (36).

Example:

8,592 #h.zéb

Number 8592 with odd numbers in blue.

Chapter 2 shows other ways of representing some color variants that may appear in the numbers.

14.5. Spaces for completion or filling in of mathematical expressions

1. Only one character to complete. There are two cases:

- a) Braille sign formed by the dots " (56) if it replaces a letter (use normal literary) or an arithmetic sign will be placed alone, since it is equivalent to a character.

For example, in an exercise where the student is asked to write the signs plus, minus, by or divided into the appropriate space to obtain the result, the spaces are replaced by the sign

" (56) as indicated in the following example:

5__4__2=10

In Braille it would be written:

#e"#d"#b=#aj



- b) If it replaces a number, the number can have one, two, three or more characters, in addition to the number sign. So, if it is a three-dimensional number numbers, you will have to write the number sign followed by " (56): #"" three times .

It must always be used with the number sign in front, since this sign (fillable character) is equivalent to a single character.

Examples:

$$12+13=_ _ \#ab+\#ac=\#""$$

Find the missing digits in the result of this operation:

$$23+145=1_ _$$

In Braille it will be written: #bc+#ade=#a""

2. Undetermined number of characters to complete.

The braille sign "-; (56-36-23)" will be used.

Example:

$$719+83=_ _ \#gai+\#hc=" -; \\ 3 \div 12 = 18 \div \square \#c@,\#ab=\#ah@," -;$$



Appendix 1. Mathematical and physicochemical units

According to what is established in this *Appendix* for the notation of units, the representation of the units of measurement of angles does not include the dot 3 at the end (see 11.5).

In the particular case of the sexagesimal and centesimal degrees, as well as the respective minutes and seconds, there will never be blank space between the numerical coefficient and the unit.

1.1. Fundamental physicochemical units

Visual sign	Braille sign	Braille dots	Meaning
m		134	metro
kg	m	13-1245	kilogram
s		234	second
K	kg	46-13	Kelvin
to	s	46-1	amp
mol	{k {a	134-135-123	mol
CD	mol cd	14-145	candle

Units of measurement are represented by leaving a blank space between the quantity and the unit, in the same way that they are represented in visual characters.

Non-breaking blank means that the quantity and unit must be on the same line.

Examples:

We are, more or less, 10 km from the city.

{we are, more or less, at
#aj km from the city.

Approximate luminosity 40 cd

{approximate luminosity
#dh cd



Although the use of a non-separation space is established for the representation of units, it will not be considered incorrect to represent it in accordance with what is established for algebraic expressions in chapters 1 and 5.

Thus, the last example would also be correctly written with the dot 5 separating the number from the first letter of the unit:

{approximate luminosity
#dh@cd

If necessary for the use of any computer program or for other reasons, You can introduce a differentiating character instead of a space. For example, point 6.

In this example, the temperature unit (kelvin) is represented by the dot 6 between the number and the symbol.

8 K #h_{k

1.2. Derived physicochemical units

For their writing, the same rules will be used as in the case of elementary physicochemical units (see 1.1).

Visual sign	Braille sign	Braille dots	Meaning
rad	rad	1235-1-145	radian
Mr	sr	234-1235	steradian
Hz	{hz	46-125-1356	hertz
N	{n	46-1345	newton
Pa	{pa	46-1234-1	pascal
J	{j	46-245	July
W	{w	46-2456	watt
C	{c	46-14	coulomb
V	{v	46-1236	volt
F	{f	46-124	farad



Visual sign	Braille sign	Braille dots	Meaning
ÿ	^w	45-2456	ohm
s	{s	46-234	Siemens
Wb	{wb	46-2456-12	weber
°C	>{c	356-46-14	degree Celsius
lm	lm	123-134	lumen
lx	lx	123-1346	lux
Bq	{bq	46-12-12345	becquerel
Gy	{gy	46-1245-13456	gray
Sv	{sv	46-234-1236	sievert

Example:

$J = \text{kg m}^2 \text{s}^{-2}$ {j=kg m1#b s1-#b

1.3. Prefixes for multiples and submultiples

Visual sign	Braille sign	Braille dots	Meaning
gives	da	145-1	deca
d		145	I said
h		125	hecto
c		14	centi
k		13	kilo
m		134	milli
M	dhckm {m	46-134	mega



Visual sign	Braille sign	Braille dots	Meaning
ÿ	'm	4-134	micro
G	{gn	46-1245	jig
n		1345	elder brother
T	{tp	46-2345	tera
p		1234	beak
P	{p	46-1234	peta
AND	{ea	46-15	exa
to		1	atto
Z	{zz	46-1356	zetta
z		1356	zepto
AND	{yy	46-13456	yotta
and		13456	yocto

The set formed by a prefix symbol and a unit symbol constitutes a new inseparable symbol.

Examples:

1 m=100 cm=0.1 km #am=#ajj cm=#j,a km

10 dm=1 m=100 cm=0.1 dam=0.001 km

#aj dm=#am=#ajj cm=
=#j,a dam=#j,jja km 1002

hPa #ajjb h{pa



Appendix 2

2.1. Ordinals

They are formed with the signs of the fifth series preceded by the numerical sign and followed by the letters "a" or "o".

Examples:

#,or 1st

#;or 2nd

#,>to 10th, etc.

The ordinals "first" and "third" are represented:

#,r 1st

#:r 3rd

2.2. Roman numerals

Although Roman numerals are represented in ink with capital letters, for their

Braille transcription will suffice to use the prefix { (46) only at the beginning of the number.

The horizontal stroke that multiplies the covered part of the number by a thousand and the double stroke which multiplies by a million, will be transcribed using : (25) and :: (25-25), respectively, behind the last affected letter.

Example:

V̄IXLDXXI {vi::xl:dxxi

2.3. Coins

The braille representation of most currency symbols is done with the sign formed by the dots 456 followed by one or more boxes representing the monetary unit. Some of them are shown in the following table:

Visual sign	Braille sign	Braille dots	Meaning
Bs	%bs	456-12-234	bolivar
ÿ	%c	456-14	colon



Visual sign	Braille sign	Braille dots	Meaning
€	%e	456-15	euro
\$	%p	456-1234	weight
\$	%s	456-234	dollar
DM	%m	456-134	German mark
¥	%y	456-13456	yen
₿	%b	456-12	bitcoin

Other signs for which 456 is not used as a prefix are:

Visual sign	Braille sign	Braille dots	Meaning
¢	^c	45-14	penny
£	@é	5-2346	pound
R\$	{r	46-1235	real

Note: In some countries, the monetary unit is written before the number and in others after this.

Examples:

\$10 = 1000¢ %p#aj=#ajjj^c

\$5 = €4.23 #e%s=#d,bc%e

R\$150 {r#aej



Appendix 3. Combinations of arrows and points

Visual sign	Braille sign	Braille dots	Meaning
↗	%a	456-1	up arrow
↘	%.	456-3	down arrow
↕	%k	456-13	up-down arrow
↖	@1	5-16	up-left oblique arrow
↗	1,	16-2	down-right oblique arrow
↘↖	@1,	5-16-2	down-right and up-left arrow
↙	@í	5-34	down-left arrow
↗	í,	34-2	up-right arrow
↖↗	@Yo,	5-34-2	down-left and up-right arrows
↙̣	@9: :o,	5-246-25	dot-Left arrow
↘̣		25-135-2	right arrow-dot
↙̣̣	'9:	4-246-25	left arrow with dot above it
↙̣̣̣	_9:	6-246-25	left arrow with dot below
↘̣̣	':either	4-25-135	right arrow with dot above it
↘̣̣̣	_:either	6-25-135	right arrow with dot below
↔̣	@:,z 5-25-2-1356		left-right arrow over the letter z
↔̣̣	@.:z 5-25-3-1356		left arrow-dot over the letter z

118

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